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# THE ANALYSIS OF MATHEMATICAL PROOF ABILITY

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**Abstract**

The ability to prove mathematically is one of the important abilities in learning mathematics. This study aims to determine the students' mathematical proof ability. This type of research is a qualitative descriptive study, which involved 38 sixth semester students of the Mathematics Education Department, Tarbiyah and Teacher Training Faculty, Alauddin State Islamic University of Makassar with 3 of them being the interview subjects. The instruments and the data collection techniques used were researchers by using diagnostic tests and interview guides. The analysis techniques used are data reduction, data display, and conclusion drawing/ verification. The results of this research show that the students' mathematical proving ability is still weak. In the indicator of the ability to construct evidence, the students are less able to make connections between known facts and the elements to be proven. Furthermore, on the indicator of the ability to read a proof, the students are less able to use definitions as a basis for providing arguments.

**Keywords**: Analysis; Ability; Mathematical Proof

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## INTRODUCTION

Becoming mathematics experts is not easy due to the huge role of mathematics in science and technologydevelopment that cannot be inseparable from the findings and research results of mathematicians in various countries which meansthat it will become difficult to achieve using mediocre abilities. A prospective mathematician needs to develop a variety of mathematical skills to build a strong foundation and initial foundation in studying mathematics into a higher level. Hartono et al., (2017: 76) argued that one of the mathematical skills that need to be possessed is the ability of mathematical proof. In line with that statement, Firmasari and Sulaiman, (2019: 2) described that mathematics as one of the learning sciences that uses reason and logic in understanding it and is closely related to believing the truth of a statement that had to be proven first

which requires the students of Mathematics Education to have the mathematicalproof ability. Syamsuri & Marethi (2018) believed that thinking mathematically in math learning of university can become a high-level thought process.

Many students still do mistakes in solving mathematical proof questions. Hodiyanto and Susiaty (2018: 130) added that mathematics learning generally conducted in various schools that is still procedural. Nurrahmah and Karim (2018: 22)stated that it just needs to follow or work on problem based on the formula and the writtenway in the reference book or it is based on lecturers' guidelines of the formula and the givenway without trying to prove the formulatruth. Meanwhile, Maya and Sumarmo, (2011: 232)argued that mathematical proof ability is an ability that must be possessed by all students who learn mathematics. It means that having mathematical proof ability can become more essential especially for students' math. By mastering the mathematicalproof ability, students are able to understand the basic concepts or the characteristics of mathematics itself.

The evidencefor mathematicians becomes the most important role. Kartika and Yazidah, (2019: 153) strengthened this statement on their argument that evidence/proof is named as a way to convince and used to test knowledge. In mathematical proof, Kartini in Firmasari and Sulaiman (2019: 2) considered that it is to make a set of arguments that are correct and logic related to which based on the inferencerules that aimed at validating the truth of a statement. Hanna in Feriyanto (2018) also believed that proof/ evidencein mathematics has a role to prove and justify. Meanwhile, the mathematical proof ability based on Lestari (2015: 130) is the ability to understand the statements or mathematics symbols and compile evidence of the truth of a statement mathematically based on definitions, principles, and theorems. The mathematical component, (Fuat, 2015) is considered as visible component because it contains several elements, namely: logic, axiomic, and formal, while there are structural components and concepts in the evidence/proof evaluation (Fuat, Nusantara, Irawan, and Irawati, 2017). Furthermore Cadwallader Olsker (2011: 34) argued that evidence/ proof is a fundamental part in mathematics, the truth of a mathematical proposition can be

known by doing the proof firstly. Studies on mathematical proof in university have been widely studied by researchers, one of them was conducted by Lee (2016) who argued that the students entering the university level must develop formal mathematics knowledge. Therefore, the students need to be trained on mathematical proof in order to comprehend the structure of formal mathematics.

Fuat, Nusantara, Irawan, and Irawati (2019)believed that evidence/ proofis one of the mathematical studies that became a trend then widely discussed by researchers in nowadays. Although mathematical proof ability is an essential and fundamental inearlier description, the student levelin mastering mathematical proof abilityis still relativelylow. Mubarok et al. (2018: 677) found that some previous researches' results revealed that the ability mathematical proof decreased along with the increasing level of material difficulties. Moreover, Firmasari and Sulaiman, (2019: 1) in their research's result concluded that the students did not understand the evidence/ proofflow and irregularities to connect every step of evidence/proof. The same cases were happened in some students when the researchers delivered lectures, especially in algebraic structure subject. Algebraic structure subjects itself are one of the mathematics courses that encourages skills of students in mathematical proof. The study results by Hartono et al., (2017: 76) also showed the mathematical proof ability of students in algebraic structure courses which reached a score of 50 of a total value of 100 that classified as still lacking.

This research is important to do based on the previous description due to the students generally were able to do mathematical proof especially at the beginning of the material. If the level of material difficulty is increasing or delivered a problem of a different type based on the example shown, students begin to experience difficulties in doing that. Ashkenazi and Itzkovitch, (2014: 186)also found that most students succeeded in using the proof method, but they did not understand the proof truth, so they failed in solving the difference problem on the same example given. Most students can do the proof method but they do not understand well the proof truth so they get failed to solve the problems on different types.

Based on the problems and research study that have been delivered previously, this research was conducted due to the aim at finding out mathematical proof ability of students. The researchers tried to analyze the level of mathematical proof ability that students have, especially in algebraic structure subject. The results of this study were expected to be a solution for them in order to overcome their difficulties in doing mathematical proof.

## METHOD

The type of this research was descriptive qualitative research which involvedthirty-eight students of semester VI of Mathematics Education Department at Faculty of Tarbiyah and Teacher Training of Alauddin State Islamic University of Makassar followed by three interview subjects. The selection of three interview subjects was based on their mathematical proof abilitythat obtained through scoring based on the indicators of mathematical proof, calculating percentages and categorizing them that can be seen onfollowing criteria:

**Table 1. Criteria for Mathematical Proof Ability**

| Percentage Interval | Criteria |
|---|---|
| $0\ \% \leq P \leq 30\ \%$ | Less |
| $30\ \% \leq P \leq 70\ \%$ | Good |
| $70\ \% \leq P \leq 100\ \%$ | Very Good |

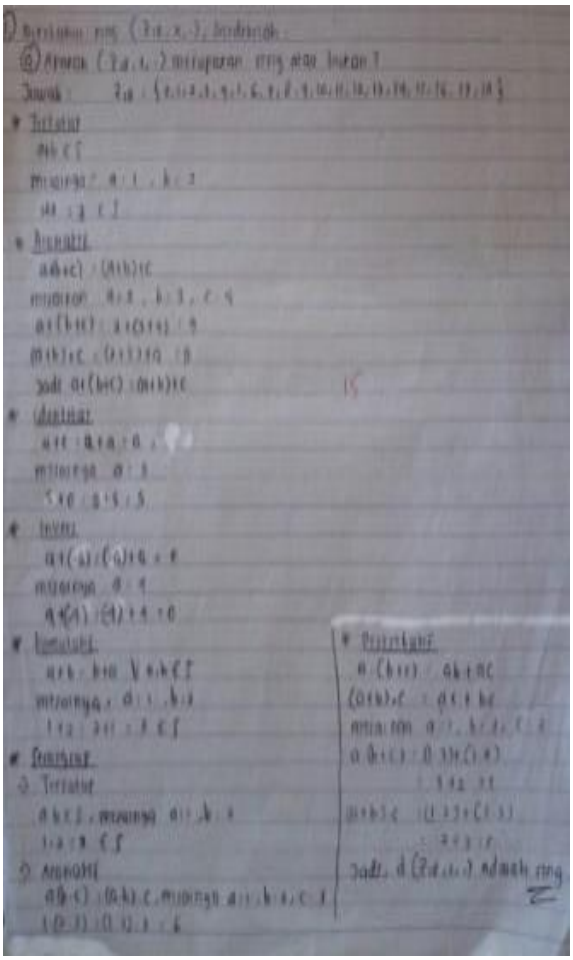
(Fadillah & Jamilah, 2016: 108)

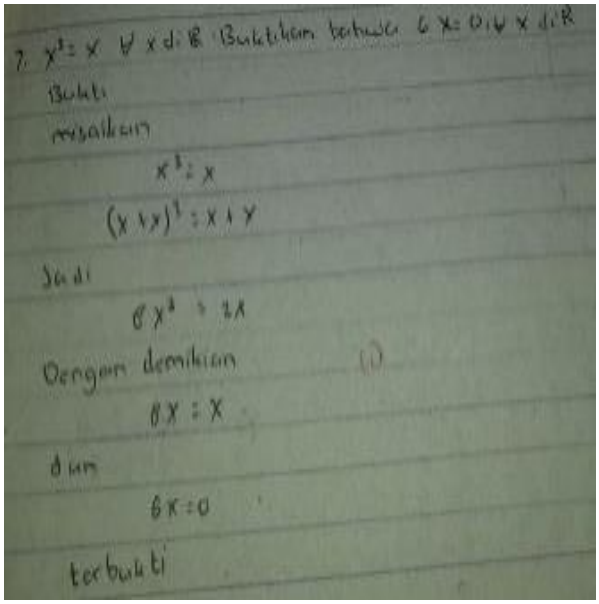
The data collection is carried out by the researchers themselves using the instruments test of mathematical proof ability (diagnostic test), and interview guidelines. The test is applied to obtain the data on students' mathematical proof abilitythat contains two indicators, namely constructing mathematical evidence and reading a proof which consisting of three questions (Selden & Selden, 2003). Furthermore, the interview guidelines consist of twenty questions items based onthe student's diagnostic test results. Diagnostic tests and interview guidelines have met valid criteria based on assessment by two validators team. Thus, the analysis techniques used were data reduction, display data, and drawing conclusion/verification (Sugiyono, 2014).

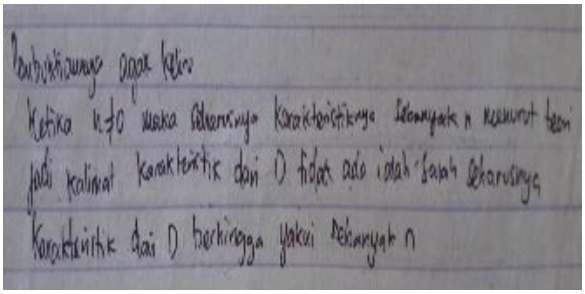
## RESULTS AND DISCUSSION

Based on this research, the ability of mathematical proof in solving problems using the ability indicators to construct evidence and the ability to read aproof is presented as follows:

**Table 2. The Results of Mathematical Proof AbilityAnalysis**

| No. | Test Results | Interview Results | Analysis |
|---|---|---|---|
| 1 |  Figure 1. A1 Subject (SA1) | Q: How do you identify of what to do in solving the problem? *J: ...kan ada biasa itu kak didalam soal yang diketahuinya disuruh ki buktikan termasuk ring atau bukan jadi saya ingat-ingat lagi bahwa teori ring itu apa-apa saja syaratnya ...* *A: ... There is actuallyin known-question that told us to prove including the ring or make me think to remember about the ring theory along with its qualifications.* Q: What do you do next? *J: ...untuk mengerjakan soal, saya pikir-pikir dulu kak baru saya tuliskan dicakaran, kalau misalnya saya yakin dengan jawaban* | Based on the results of that test and interviews it is known that SA1 studentis able to identify the following question to find out about what he/she doesnext by remembering the qualification. However, SA1 chose to write down an example in proving the traits that must be fullfilled. |

| No. | Test Results | Interview Results | Analysis |
|---|---|---|---|
| | | *saya baru saya tuliskan kembali dalam kertas jawabansaya*<br>*A: ... in answering the questions, I think it first, then i try to calculate it in notes/ a rough copy, then if i convinced with the answer on that notes, then i rewrite on my asnwer sheet.* | |
| 2 |  <br>Figure 2. A2 Subject (SA2) | Q: Can you please explained deeply on question number two?<br>*J:kalau yang nomor 2 kak, saya pake pemisalan berdasarkan definisi yang diberikan pada soal. Kemudian saya kaitkan dengan yang mau dibuktikan.*<br>*A: on question number two, i applied some examples based on the given definition on that question. Then I try to associate it with what I want to prove.* | It is shown that SA2 students on his settlement due to question number two, it is found some appropiate steps given which then delivered wrong calculation 6×=0 |

| No. | Test Results | Interview Results | Analysis |
|---|---|---|---|
| 3 |  Figure 3. A3 Subject (SA3) | Q: How do you make prediction? *J: ...biasanya saya ambil kontradiksinya* *A: I usuallydo contradiction* Q: How do you determine the truth or errorof evidence/ proof? *J:...,saya identifikasi dulu apakah pernyataan yang disajikan sudah benar sampai kebawah atau salah.* *A: ..., I begin with the identification at the beginning statement till the end whether it is presented correctly or wrongly.* | Based on the test results and interviews it is known that SA3 student is able to provide arguments in evidence. Eventhough, the evidence/ proof steps are not clearly shown. |

On table 2, it is known that the proof in solving the problem was still inexact. In the first indicator of constructing mathematical proof using evidence methods and facts, concepts, and mathematical principleswhich the students were able to organize and manipulate the facts to show evidence and they wereable to use definitions, premises, or theorems related, but they could not make a connection between known-facts and elements to be proven. It means that they, the students' mathematical proof ability was still weak. Maya and Sumarmo (2011: 234) justified that students' mathematical proof abilityare related to mathematical comprehension skills. Salsabila, (2019: 46) added that understanding prerequisite concepts and mathematical communication skills have positive and significant influence on students' mathematical proof ability by

69.3%. Nurrahmah & Karim, (2018: 28) tried to conclude that students have difficulty in solving the evidencequestions due to the weak concept of prerequisite materials. Some factors that caused low mathematical proof ability of students based on Angraini et al., (2019: 196) and Perbowo & Pradipta (2017: 81) who stated that they are not used to doing mathematics proof and have less understanding about mathematics proof.

In the second indicator of mathematical proof ability, namely the ability to read a proof to determine the truth or its error by looking at the suitability between the axiom system, the premise or the existing theorem and complete the proof, theywere able to study a mathematical statement in order to determine the truth or to show the errors of the statement by delivering a hypothesis based on the pattern and nature of some statements that had been obtained, but they could not deliver the definition as the basis of providing their arguments. Kartika dan Yazidah, (2019: 152) justified that statement that there are still some students who use sampling technique to do direct evidence. Mahfudy, (2017: 31) believed that there are two types of proof in mathematics, namely syntactic and semantic proof production. Semantic proof tends to use intuition to deliver assumptions and doreasoning assumptions made to help facilitating the evidence. Utari dan Hartono, (2019: 11)on their research argued that the content of reasoning and mathematical proof on the evaluation question givenwhich has not fully provided students an opportunity to make reasoning and evidencein answering the question. So in improving mathematical proof, Agustyaningrum et al., (2020: 832) argued that it is necessary to practice reasoning and mathematical proof and provides the easy way for them to comprehend deeply about the teaching materials.

## CONCLUSION

Based on this research, it is concluded that students' mathematical proof ability was still weak. In ability indicators to construct evidence, students could not connect between the known-facts and the elements that was proven. Furthermore, in the reading ability indicator of a proof, students could not use the definition as the basis in delivering their arguments.

## REFERENCES

Agustyaningrum, N., Husna, A., Hanggara, Y., Abadi, A. M., & Mahmudi, A. (2020). Analysis of mathematical proof ability in abstract calgebra course. Universal Journal of Educational Research, 8(3), 823–834. https://doi.org/10.13189/ujer.2020.080313

Angraini, L. M., Sundawan, M. D., & Noto, M. S. (2019). Analisis Proses Berpikir Menyusun Bukti Matematis Mahasiswa Calon Guru Pada Mata Kuliah Struktur Aljabar. Euclid, 6(2), 189. https://doi.org/10.33603/e.v6i2.2564

Applebaum, D., & Applebaum, D. (2010). Proof by mathematical induction. Probability and Information, 1(3), 247–248. https://doi.org/10.1017/cbo9780511755262.014

CadwalladerOlsker, T. (2011). What Do We Mean by Mathematical Proof? Journal of Humanistic Mathematics, 1(1), 33–60. https://doi.org/10.5642/jhummath.201101.04

Fadillah, S., & Jamilah, J. (2016). Pengembangan Bahan Ajar Struktur Aljabar Untuk Meningkatkan Kemampuan Pembuktian Matematis Mahasiswa. Jurnal Cakrawala Pendidikan, 1(1), 106–113. https://doi.org/10.21831/cp.v1i1.8379

Feriyanto. (2018). The Ability of Students' Mathematical Proof in Determining the Validity of Argument Reviewed from Gender Differences. Journal of Physics: Conference Series, 947(1). https://doi.org/10.1088/1742-6596/947/1/012042

Firmasari, S., & Sulaiman, H. (2019). Kemampuan Pembuktian Matematis Mahasiswa Menggunakan Induksi Matematika. Journal of Medives : Journal of Mathematics Education IKIP Veteran Semarang, 3(1), 1. https://doi.org/10.31331/medivesveteran.v3i1.642

Fuat. (2015). Improving learning to increase ability to prove geometry in. BecomingReflective Educators and Professionals of Learning.

Fuat, Nusantara, T., Irawan, E. B., & Irawati, S. (2017). A case study on students' proof construction by using (H→H1)∧(¬C→¬H1)→C against the proof. Building National Characters through Education for Noble Civilization.

Fuat, Nusantara, T., Irawan, E. B., & Irawati, S. (2019). Students' mathematical conviction in Mathematics proof. IOP Conference Series: Earth and Environmental Science, 243(1). https://doi.org/10.1088/1755-1315/243/1/012133

Hodiyanto, H., & Susiaty, U. D. (2018). Peningkatan Kemampuan Pembuktian Matematis Melalui Model Pembelajaran Problem Posing. MaPan, 6(1), 128–137. https://doi.org/10.24252/mapan.2018v6n1a12

Kartika, E. D., & Yazidah, N. I. (2019). Analisis Kemampuan Pembuktian Matematis Pada Matakuliah Analisis Real Berdasarkan Adversity Quotient. Prima: Jurnal Pendidikan Matematika, 3(2), 152. https://doi.org/10.31000/prima.v3i2.1385

Lee, K. S. (2016). Students' proof schemes for mathematical proving and disproving of propositions. Journal of Mathematical Behavior, 26–44.

Lestari, K. E. (2015). Analisis Kemampuan Pembuktian Matematis Mahasiswa Menggunakan Pendekatan Induktif-Deduktif Pada Mata Kuliah Analisis Real. MENDIDIK: Jurnal Kajian Pendidikan Dan Pengajaran, 1(2), 128–135.

Mahfudy, S. (2017). Strategi Pembuktian Matematis Mahasiswa Pada Soal Geometri. JTAM | Jurnal Teori Dan Aplikasi Matematika, 1(1), 31. https://doi.org/10.31764/jtam.v1i1.101

Matematika, J., Matematika, F., Ilmu, D. A. N., & Alam, P. (2017). Melalui Model Pembelajaran Discovery. 76–85.

Maya, R., & Sumarmo, U. (2011). Mathematical Understanding and Proving Abilities : Experiment With. Journal On Mathematics Education (Indo MS-JME), 2(2), 231–250.

Mubarok, M. S., Pujiastuti, E., & Suparsih, H. (2018). Meningkatkan Kemampuan Pembuktian Matematis dan Rasa Ingin Tahu Siswa Kelas XI MIPA SMA Negeri 6. PRISMA, Prosiding Seminar Nasional Matematika, 1(2000), 677–683.

Nurrahmah, A., & Karim, A. (2018). Analisis Kemampuan Pembuktian Matematis Pada Matakuliah Teori Bilangan. JURNAL E-DuMath, 4(2), 21. https://doi.org/10.26638/je.753.2064

Perbowo, K. S., & Pradipta, T. R. (2017). Pemetaan Kemampuan Pembuktian Matematis Sebagai Prasyarat Mata Kuliah Analisis Real Mahasiswa Pendidikan Matematika. KALAMATIKA Jurnal Pendidikan Matematika, 2(1), 81. https://doi.org/10.22236/kalamatika.vol2no1.2017pp81-90

Salsabila, E. (2019). Influence of prerequisite concepts understanding and mathematical communication skills toward student's mathematical proving ability. Pythagoras: Jurnal Pendidikan Matematika, 14(1), 46–55. https://doi.org/10.21831/pg.v14i1.25067

Selden, A., & Selden, J. (2003). Validations of Proofs Considered as Texts : Can Undergraduates Tell Whether an Argument Proves a Validations of Proofs Considered as Texts : Can Undergraduates Tell Whether an Argument Proves a Theorem ?January.

Sugiyono. (2014). Metode Penelitian Kuantitatif, Kualitatif, dan R&D. Alfabeta.

Syamsuri, S., & Marethi, I. (2018). APOS analysis on cognitive process in mathematical proving activities. International Journal on Teaching and Learning Mathematics, 1(1), 1. https://doi.org/10.18860/ijtlm.v1i1.5613

Utari, T., & Hartono, H. (2019). Muatan penalaran dan pembuktian matematis pada buku teks matematika SMA kelas X Kurikulum 2013. Jurnal Riset Pendidikan Matematika, 6(1), 1–13. https://doi.org/10.21831/jrpm.v6i1.17002