STUDENT THINKING PROCESS IN SOLVING MATHEMATICAL PROBLEMS BASED ON APOS

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Abstract

This study aimed to provide insights into student thinking process in solving factorization of algebraic expressions based on APOS. A qualitative-exploratory design was employed in this study, where the participants were asked to solve mathematical problems and perform think-aloud protocols. Interview was conducted to extract more interesting data and clarify the participants’ answers and logical analysis. The research subject was a student of class VIII of SMPN 23 Ambon which two students with the initials S1 and S2. The results showed that the thought process of students in solving the problem of factorization of algebraic tribes based on the APOS framework can be classified into 2 categories. The first category is complete thinking structure that was indicated by providing the correct answer to the problem. This process began with interiorization as Action, followed by coordination, encapsulation, reversal, regulation or de-encapsulation as Process towards the object. The second category or scheme is incomplete thinking structure, marked by making errors during the problem-solving process and fixing the errors after reflection. The incomplete thinking structure occurred on interiorization as Action and coordination, reversal and de-encapsulation as Process towards the object. The study found only two categories (complete and incomplete thinking structures), so it is possible for future research to find other categories.

Keywords: thinking process; mathematical problem solving; APOS.

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INTRODUCTION

Mathematics learning is an activity that requires a thinking process. A person who thinks is doing mental activities. People who learn math perform mental activities through problem solving. However, mathematics learning in Indonesia often puts an emphasis on memorizing concepts. This certainly has a significant effect on the students’ thinking process. Based on the TIMSS four-year survey in 2015, mathematical abilities of Indonesian students ranked 45 out of 51 participating countries (Reddy et al., 2016: 21). Meanwhile, a three-year study
conducted by PISA in 2015 showed that Indonesian students scored among the lowest (62 out of 70 countries) in mathematics (OECD, 2019). Therefore, mathematics research needs to focus more on students’ thinking processes and development of math problems based on PISA and TIMSS problems, as has been researched by (Ahyan, Zulkardi, & Darmawijoyo, 2014); (Oktiningrum, Zulkardi, & Hartono, 2016); (Noviarsyh Dasaprawira, Zulkardi, & Susanti, 2019). Vincent Ruggiero defines thinking as “a mental activity carried out to solve a problem, make a decision and fulfill curiosity in understanding a problem” (Sopamena, 2017). Thinking process can only be recognized when a person takes action through problem solving. Ed Dubinsky in his research on the construction process of "genetic decomposition" suggests that one’s actions in solving mathematical problems can be seen from interiorization and encapsulation, hereinafter referred to as APOS (Action, Process, Object, and Schema) (Dubinsky, 2014); (Arnon et al., 2014). This study employed APOS approach to describe students’ thinking process in solving factorization of algebraic terms.

Until the present, studies on the application of APOS approach have only focused on two aspects. There are various studies that examine learning by applying the APOS framework (Salgado & Trigueros, 2015); (Borji, Alamolhodaei, & Radmehr, 2018); (Arnawa, Yerizon, & Nita, 2019); (Arnawa, Yerizon, Nita, & Putra, 2019); (Afgani, Suryadi, & Dahlan, 2019). In addition, a number of research has been carried out to assess students’ mathematical abilities and understanding using the APOS framework (Tziritas, 2011); (Maharaj, 2013); (Mena-Lorca & Parraguez, 2016); (Bansilal, Brijlall, & Trigueros, 2017); (Afgani, Suryadi, & Dahlan, 2017); (Hidayatullah, 2019). Salgado & Trigueros point out that by using an activity model designed from genetic decomposition, although a lot of literature and teachers say it is difficult, at least three students from the researched group succeeded in demonstrating the construction of eigenvalues and the concept of eigenvectors. Furthermore, Bansilal, Brijlall, & Trigueros suggest that the best way to help students develop positive attitudes towards mathematics is to facilitate their understanding of mathematics. From these two aspects, there has been no specific
study looking at how students think in solving mathematical problems using the APOS framework.

Many themes have been discussed in past studies related to the APOS framework, including: the APOS framework itself (Dubinsky & Mcdonald, 2005); (Cetin & Dubinsky, 2017), genetic decomposition (Dubinsky, 1991), APOS learning and mathematical understanding according to the APOS framework (Asiala et al., 1996).

Piaget explains that in solving a mathematical problem, an individual always goes through a reflective abstraction construction process that is exemplified in logical-mathematical thinking at an early age (Dubinsky, 1991). The construction processes of reflective abstraction include: interiorization, coordination, encapsulation, generalization and reversal construction (Dubinsky). The main mental mechanisms in building mental structures of action, processes, objects and schemes are called interiorization and encapsulation (Dubinsky, 2014); (Arnon et al., 2013). Action is a transformation of reactions to stimuli that are felt by individuals externally (Asiala et al., 1996); (Bansilal, 2014); (Dubinsky & Mcdonald, 2005). Process is a mental structure in carrying out the same operations as action, but entirely occurs in the individual mind (interiorization occurs) (Dubinsky & Mcdonald, 2005); (Bansilal, 2014); (Asiala et al., 1996). Object is the totality of transformation consciousness encapsulated into cognitive objects (Arnon et al., 2014); (Dubinsky, 2014); (Arnon et al., 2013).

Furthermore, this reflective abstraction is transformed into mental object construction and mental action, hereinafter referred to as Scheme (Dubinsky, 2002: 101). Piaget (1952, p. 7) defines Scheme as components of cohesive, repetitive action that have components of action that are closely related and governed by core meanings. Scheme is a collection of objects and processes that are more or less coherent (Dubinsky, 2002: 101). Mathematical topics often involve many actions, processes, and objects that need to be organized and linked into a coherent framework, called a Scheme (Arnon et al., 2014). Thus, a Schema is a structured group of concepts than can be used to represent objects, scenarios or sequences of events or relationships. Therefore, this study will describe students’ thinking
process in solving mathematical problems based on the APOS framework. The framework used in this study referred to the genetic decomposition process.

METHOD

This study was conducted in SMP N 23 Ambon, a public junior high school in Ambon. There are 13 teachers and 3 study groups that could potentially participate in this study. This study attempted to reveal the occurrence of students' thinking processes in solving mathematical problems. In this study, the participants were asked to solve problems by expressing aloud what they were thinking (Think Aloud protocols). The participants' thinking processes were then assessed based on the APOS framework (Action, Process, Object and Scheme). The data were collected in the form of verbal data, so this study was categorized as a qualitative-exploratory study. The instruments used in this research are the researchers themselves as the main instruments and other supporting instruments, among others: student worksheets, interview guidelines, field records, and documentation.

The selection of the study participants was based on the students' ability to solve problems instinctively. This article describes the problem solving process of a subject who provided a perfect or correct answer (S1) and from a subject who provided a wrong answer, but was then given the opportunity to reflect on his answer so that he could correct his answer (S2). The mathematical problem administered to the students was factorization of algebraic terms.

The data of this study were collected to describe the participants’ thinking process in solving representative mathematical problems based on the APOS framework. The stages of data collection consisted of the following activities:

1. A direct observation was conducted while the participants were completing the task.
2. The participants or the subjects were assigned to solve a problem on factorization of algebraic expressions and asked them to verbally express what they were thinking during the problem-solving process.
3. The participants’ verbal expressions were recorded.
4. Interview was done to explore what the participants were thinking during the problem-solving process.

The verbal and written data were analyzed for consistency. A clarification interview was conducted if there were inconsistent data. The clarification interview was conducted by re-interviewing the participants. If the data remained inconsistent, they would be omitted from the analysis.

Data analysis was performed in several stages that included: (1) transcribing the verbal data, (2) examining all available data from various sources, namely the think aloud data, the results of the interviews, the results of the observations that had been written in the field notes, and the results of student concept construction, (3) reducing the data set obtained based on the APOS framework by making a core summary, process, and statements that need to be kept in it, (4) compiling data in units and categorizing them by coding, (5) describing the students thinking structure in solving the problem on factorization of algebraic expressions based on APOS, (6) analyzing the students’ thinking process based on APOS, (7) analyzing interesting phenomena and (7) drawing conclusions (Creswell, 2012: 238-241).

RESULT AND DISCUSSION

1. Analysis of the S1 and S2 Thinking Process based on the APOS Framework

Subject 1 (S1) demonstrated a complete cognitive structure and met the criteria of the APOS framework when solving the factorization of algebraic expression task. This means that in the problem solving process, S1 carried out a complete thinking process. S1 immediately did a reflection by questioning his problem-solving result. Furthermore, the reflection result was also true. On the other hand, in completing the task, S2 showed an incomplete cognitive structure and a disconnected substructure, but the process still met the criteria of the APOS framework. In addition, it can also be concluded that S2 made a mistake in the process of completing the task. However, after being given the opportunity to reflect, he replaced his answer with the correct answer. This means that there are students who have a complete thinking structure marked by a correct problem-solving process and there are students who have an incomplete thinking structure.
(the problem solving process is wrong but after doing a reflection, the answer is correct). This finding corroborates with those of Subanji (Subanji & Nusantara, 2016). The S1 and S2 thinking processes are described in detail as follows.

a. **S1 Thinking Structure**

When S1 was asked to solve an essay question, he could understand the problem and provide the right solution. Then, S1 was given the opportunity to reflect on the test question. The results of the S1 reflection were also proven correct, so the answers obtained from before and after the reflection were basically the same, namely the correct answer. S1 thinking structure can be seen in Diagram 1 and Diagram 2.

![Diagram 1. Structure of thought S1 before reflection](image-url)
Diagram 2. Structure of thought S1 after reflection

Diagram 2 shows that S1 has constructed a complete thinking scheme, suggesting that there was no error done during his thinking process. The answer provided by S1 before and after the reflection was correct.

b. S2 Thinking Process

When S2 was asked to solve the task, the first thing that S2 did was to read the question, then explain the purpose of the question, and solve the question immediately. Similar to S1, S2 was also given the opportunity to reflect on the answer. After reflection, S2 realized that he had made a mistake and thus he made
an attempt to correct the answer. The S2 thinking structure can be seen in Diagram 3 and Diagram 4.

Diagram 3. Structure of thought S2 before reflection
Diagram 4. Structure of thought S2 after reflection

The diagram above shows that S2 reconstructed his thinking scheme when he detected an error in it and quickly accommodated his thinking scheme. S2’s thinking scheme accommodation occurred when S2 took the child’s age into account. It was at this stage that S2 made a mistake. Furthermore, the accommodation of S2 thinking scheme also occurred when S2 used the properties of operations to solve the algebraic term operations. S2 made a mistake in the process of adding a certain number. After being given the opportunity to reflect, S2 was able to correct the error and obtain the correct answer.

2. Analysis on S1 and S2 Problem-Solving Processes based on the APOS Framework

a. S1 Thinking Process in Solving a Problem on Factorization of Algebraic Expressions based on the APOS Framework

S1 demonstrated a complete thinking structure when solving a problem in the form of an essay test on factorization of algebraic terms. This was marked by
the absence of errors in the problem solving process before and after reflection. S1 thinking process can be seen as in Figure 1.

Figure 1. S1’s Answer before and after Reflection

In completing the task, S1’s thinking process began with contemplating the problem (interiorization) by determining what is known and what is asked on the test. Next, S1 translated the problem in his mind and put it onto the answer sheet. This explains that at this stage, S1 did Action. This finding is also reinforced by the S1’s statement during the interview, as shown below.:

P : Do you understand the problem?
S1 : Yes I do. It is explained that five years ago, the total age of mother and her twin daughters was 40, given the fact that the age of mother was 30 years. I was asked to determine the age of each twin individual.

Then, S1 assumed that x was the age of mother, which is 30 years old and y was the age of each twin individual and 2y was the total age of the twins. What S1 did suggests that he performed encapsulation that is a mental transformation from Process to Object. In other words, S1 was doing Process.

Finally, S1 obtained the equation $x + 2y = 40$, indicating that he had done generalization or de-encapsulation (Action and Process). In addition, S1 substituted x into the equation $30 + 2y = 40$ and did interiorization as Action. Then, S1 performed a mental construction from coordination construction Process.
and reversal towards the equation by applying the algebraic operation property, which is $2y = 40 - 30$. Then, S1 did encapsulation as **Process**, that is $2y = 10$ followed by $y = \frac{10}{2}$ and $y = 5$. Therefore, S1 was doing **Object**.

The final stage of S1’s thinking process was drawing a conclusion and doing a reflection. S1 did the reflection consciously without being asked. However, he found no error and thus was convinced about the answer. S1 produced **Object** from **Action** and **Process** and organized the **Object** into a **Scheme**. It thus suggests that at this stage, S1 performed **Scheme** (Sopamena, Nusantara, Irawan, & Sisworo, 2016). This means that the students successfully carried out the completion process using the characteristics of APOS, as stated in (Syamsuri, Purwanto, Subanji, & Irawati, 2017).

**b. S2 Thinking Process in Solving a Problem on Factorization of Algebraic Expressions based on the APOS Framework**

Subject 2 (S2) showed an incomplete thinking structure based on the APOS framework in solving the factorization of algebraic expression task. The incomplete thinking structure was marked by an error made by the subject during the problem-solving process. Then, S2 did a reflection and finally could provide the correct answer and perform thinking process that meets the criteria of APOS. The S2’s answer to the problem can be seen in Figure 2.

![Figure 2. S2’s Answer before Reflection](image-url)
S2 began the problem-solving process with interiorization and association by writing down what is known from the question. The error made by S2 was to assume $x$ as the age of mother and $2y$ as the age of the twin daughters.

Then, S2 did encapsulation that is a mental transformation from Process to Object, meaning that S2 performed Process. This finding is also supported by S2’s statement as follows.

\[ P : \text{Can you tell me what is being asked in the problem?} \]

\[ S2: \text{The question is to figure out the age of each twin individual.} \]

Based on his understanding, S2 obtained the equation $x + 2y = 40$. It means that S2 performed Action and Process towards Object through interiorization and coordination. Then, S2 substituted $x$ into the equation to obtain $30 + 2y = 20$ (reversal). This means that S2 carried out Action and Process towards Object through interiorization, coordination, reversal, encapsulation and generalization (de-encapsulation) (Dubinsky, 1991). After that, S2 did Action and Process of coordination construction of $30 - 20 = 10$ and $2y = \frac{10}{2}$ to encapsulate $y = 5$ into Object. However, an error was detected. S2 should have obtained the equation $30 + 2y = 40$, $2y = 40 - 30$ so $2y = 10$, then $2y = \frac{10}{2}$ and $y = 5$. Therefore, S2 did a reflection and successfully obtained the correct answer. In an interview, S2 provided the following statement:
$P$ : Are you convinced about your answer?

$S2$ : hmm, (quiet while thinking)

$P$ : Try checking your answer.

$S2$ : (while examining the answer), Oh I forgot to write what has been stated in the problem. I just assumed the value using a symbol. My second error was found in the equation. Instead of $30 + 2y = 20$, the equation should be written $30 + 2y = 40$ and $2y = 40 - 30$, $2y = 10$ hence $y = 5$.

$P$ : Are you sure about the answer?

$S2$ : Yes, I’m sure about it.

$P$ : Don’t you want to re-examine your answer?

$S2$ : (examining the answer), no, I’m sure this time.

Based on the participant’s answers and the results of the interviews above, it was known that $S2$ made modifications to his thinking process. $S2$ performed Action and Process towards Object through interiorization of the problem and interiorization and coordination of the equation. Next, $S2$ performed encapsulation, reversal, and generalization (de-encapsulation) so that he could obtain the correct answer (Dubinsky, 1991). This means that $S2$ has a complete or correct modification of the thinking scheme. This means that the students successfully carried out the completion process using the characteristics of APOS, as stated in (Syamsuri, Purwanto, Subanji, & Irawati, 2017).

CONCLUSION

Based on the results of the previous research and discussion, several conclusions were obtained as follows: (1) there are differences in the students’ mathematics learning outcomes who are taught with NHT-type cooperative learning model with the students who are taught using the direct learning model in class X ISS MA Muhammadiyah Bantaeng. (2) there is a significant difference between the results of mathematics learning in control and experimental classes with high student learning interests. (3) there is a significant difference between the mathematics learning outcomes in control and experimental classes with low students’ learning interests. (4) there is no interaction with the use of NHT type
cooperative learning model with the learning interest toward the students’ mathematics learning outcomes in class X ISS MA Muhammadiyah Bantaeng.

Suggestions that can be given after doing this research is that mathematics teachers are expected to teach by applying learning models that can increase students’ interest in participating in the learning process.

REFERENCES


