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STUDENTS' GEOMETRY THINKING ON CIRCLE MATERIAL BASED ON VAN HIELE'S THEORY

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Abstrak

Tujuan penelitian ini adalah untuk mendeskripsikan berpikir geometri siswa pada materi lingkaran berdasarkan teori Van Hiele (Visualisasi, Analisis, Deduksi Informal, Deduksi, Rigor). Subjek penelitian ini adalah 3 siswa kelas IX-A SMP Hang Tuah 4 Surabaya berdasarkan kemampuan awal matematis (tinggi, sedang, dan rendah). Data kemampuan awal matematis diperoleh dari data nilai raport semester genap tahun akademik 2021-2022 dan berdasarkan pertimbangan guru matematika kelas IX-A. Jenis penelitian ini adalah penelitian kualitatif deskriptif. Pengumpulan data dalam penelitian ini menggunakan teknik tes dan wawancara. Instrumen tes yang digunakan dalam penelitian ini adalah tes geometri lingkaran berupa soal-soal matematika dengan materi lingkaran yang disusun berdasarkan indikator van Hiele tanpa tahap 4 (Rigor) dengan pertimbangan bahwa penelitian dilakukan pada kelas IX SMP jadi belum mampu memahami materi pada tahap 4 (rigor). Siswa dengan kemampuan awal matematika tinggi dan sedang kemampuan berpikir geometrinya sudah berada pada tahap 2 (deduksi informal) sedangkan siswa dengan kemampuan awal matematika rendah, kemampuan berpikir geometrinya berada pada tahap 0 (visualisasi).
Kata kunci: Berpikir Geometri; Lingkaran; Teori Van Hiele

Abstract

The purpose of this study was to describe students' geometri thinking on circle material based on Van Hiele's theory (Visualization, Analysis, Informal Deduction, Deduction, Rigor). The subjects of this study were 3 students of class IX-A SMP Hang Tuah 4 Surabaya based on their initial mathematical abilities (high, medium, and low). The initial mathematical abilities data was obtained from data on report for the even semester of the 2021-2022 academic year and based on the considerations of the mathematics teacher for class IX-A. This type of research is descriptive qualitative research. Collecting data in this study used tests and interviews. The test instrument used in this study was a circle geometry test in the form of mathematical questions with circle material arranged based on the Van Hiele indicator without stage 4 (Rigor) with the consideration that the research was conducted in class IX SMP so students have not been able to understand the material at stage 4 (rigor). Student with high and medium initial mathematical abilities, geometric thinking abilities is already at stage 2 (informal deduction) while student with low initial mathematical ability, geometric thinking ability is at stage 0 (visualization).
Keywords: Geometry Thinking; Circle; Van Hiele Theory

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INTRODUCTION

One of the branches in mathematics is geometry. Geometry is closely related to the problems that exist in everyday life. Geometry material includes the relationship between lines, angles, triangles and quadrilaterals, the Pythagorean theorem, circles, cubes, blocks, prisms, pyramids, and their nets, similarity and congruence, tubes, cones, and spheres (Nuansari, 2016). Geometry is said to be a visualization of objects on earth (Cesaria, Herman, & Dahlan, 2021), square tiles, circular rings, conical birthday hats, and many others. Geometry touches all aspects of life (Mamolo, Ruttenberg-Rozen, & Whiteley, 2015). Geometry is very important because geometry makes humans understand the world and its contents, geometry helps humans in developing problem solving abilities, geometry helps humans understand other branches of mathematics because geometry plays a major role in mathematics, geometry helps humans in everyday life, geometry helps humans in solving puzzles. -puzzle and fun (Khoiri, 2014).

Geometry is very important therefore students' geometric thinking ability need to be improved. Geometric thinking ability is the ability of students to observe, recognize, build a definition of an object and then be able to solve problems about the object (Idlal & Suyitno, 2022). Geometric thinking ability is the ability to solve problems or questions related to mathematics based on ability indicator (Salasiwa, 2021). The ability to think geometry helps students develop reasoning power and find solutions in solving problems related to geometry (Putri & Nopriana, 2019). Knowing students' geometric thinking processes is very important for teachers to be able to find out the location and types of errors made by students in solving geometric problems. The theory that explains the students' geometric thinking process is Van Hiele's theory (Cesaria et al., 2021). Van Hiele theory is a theory about the level of thinking of students in studying geometry, one of which is in flat

shapes, where students will not go up to the upper or higher level without passing the previous level or lower level (Musa, 2018; Sahara & Nurfauziah, 2021).

Many studies have been carried out on Van Hiele's level of geometric thinking. Some of them are research to determine students' geometric thinking stages based on Van Hiele's theory in terms of cognitive style (Hidayat, Zubaidah, & Mirza, 2015; Muhassanah & Mulyatna, 2020), in terms of gender differences (Amalliyah, Dewi, & Dwijanto, 2021; Musa, 2018), and in terms of self-efficacy (Idlal & Suyitno, 2022). In this study, an analysis will be carried out to determine the stages of students' geometric thinking based on their initial mathematical ability on the subject matter of circle geometry. The circle material was chosen because the application of the circle concept is often encountered in everyday life (Rosita, et al., 2020), besides that the circle material is also a prerequisite material to be able to understand the concept of curved side space. There is a relationship between the mastery of the circle material on students' spatial abilities in the curvature material (Rikanah & Widodo, 2016).

Early mathematical ability was chosen as an additional variable in this study because early mathematical ability can affect students' ability to solve a mathematical problem. A mathematical problem can be solved if students can relate what students already have in their thinking structure (initial abilities) in the form of mathematical concepts, with the problems students face (Akramunnisa & Sulestry, 2016). Based on the description above, the stages of geometric thinking and early mathematical abilities are factors that differ from one student to another so that it needs to be considered in the learning process. Therefore, it is necessary to conduct research to determine the stages of students' geometric thinking on circle material based on Van Hiele's theory based on initial mathematical ability. The purpose of this research is to describe students' geometric thinking on circle material based on Van Hiele's theory based on high, medium, and low initial mathematical abilities.

METHOD

This type of research is descriptive qualitative research. This research was conducted at Hang Tuah 4 Junior High School Surabaya with the research subjects being grade IX-A students. The research subjects were selected as many as three students based on initial mathematical abilities, one student with high initial mathematical ability, one student with medium initial mathematical ability, and one student with low initial mathematical ability. Data initial mathematical ability was obtained from even semester report for the 2021-2022 academic year and consideration of the mathematics teacher for grades IX-A.

Collecting data in this study used technical tests and interviews. Data collection in this study used two types of instruments, namely the main instrument and the auxiliary instrument. The main instrument was the researcher, who collect data directly from data sources. The researcher as the main instrument interacts directly with the students who were selected as research subjects. The auxiliary instrument in this research was a test instrument designed to collect written data about the ability to think geometrically on circular material based on Van Hiele's theory.

The circle geometry test instrument was in the form of mathematical questions with circle material arranged based on indicators for each Van Hiele stage. The question indicators used to compile the test instrument can be seen in Table 1.

Table 1. The Question of Indicator

The Van Hiele Stage	Indicator
Visualization	Identify a circle based on its overall appearance in a simple drawing.
Analysis	Identify the elements of a circle and their relationships.
Informal Deduction	Identify and use strategies or deep thinking to solve problems.
Deduction	Prove axiom-setting relationships described informally at second level.

The Van Hiele thinking stage indicator on the circle material was only presented up to stage 3 (deduction) with the consideration that the research was conducted in class IX of junior high school so students have not been able to

understand the material at stage 4 (rigor). This study used source triangulation, namely by checking or comparing the suitability of data obtained from test results with interview results.

RESULT AND DISCUSSION

The subjects of this study were three junior high school students with different abilities (high, medium, and low). These high, medium, and low abilities are based on the previous semester's report. Students' initials and mathematical abilities can be seen in Table 2 below.

Table 2. Student Initials and Mathematical Ability

Inisial	Mathematical Ability
TSK	High
DAP	Medium
STA	Low

Subjects with High KAM (TSK)

At stage 0 (visualization), question number 1 TSK could determine which was a circle and which was not a circle. TSK already understands the concept that a circle was a flat shape that only had two dimentional space, length and width. The tube was a shape that had three dimensional space, length, width, and height. Because of this understanding, TSK believed that the piggy bank was not a circle but a tube that had a circle base and lid. This shows that TSK can distinguish between two dimentional space and three dimentional space. It is indicators of stage 0 (visualization) where at this stage students can identify circles in a set of other geometry objects (Nuansari, 2016). The following is the researcher's interview with TSK.

P : Is it just a wall clock, coins, bottle caps, and buttons?
TSK : Ehhhm… (the subject looks back at the picture in the question). The ring is also a circle, the plate is too, the wheel is also ma'am.
P : Then the rest? The other pictures are not circles?
TSK : No. This piggy bank with the lid and bottom is indeed a circle, but if the whole it is a tube, not a circle.
P : Oh I see. So when asked whether this piggy bank is a circle or not, the answer is no. But when asked what shape the lid is on the piggy bank, the answer is circle. Oh, I see?

TSK : Yes, Maam.
P : If so, what shape is this piggy bank?
TSK : Tube
P : Is the tube flat or spaced?
TSK : Three dimensional space
P : What is the difference between two dimensional space and three dimensional space?
TSK : Two dimentional space has length and width, if three dimentional space is length, width, and height.

In stage 2 (informal deduction) TSK already understood the relationship between radius and diameter, that the length of the radius is half the length of the diameter. However, TSK was less precise in converting units from cm^2 to m^2 as shown in Figure 1.

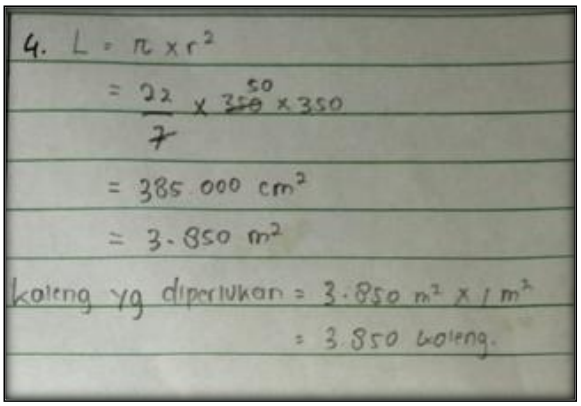

Figure 1. TSK's Answer to Question Number 4

The translation of Figure 1 is shown in Figure 2.

$L = \pi r^2$

$= \frac{22}{7} \times 350 \times 350$

$= 385.000\ cm^2$

$= 3.850\ m^2$

The required cans= 3.850 m^2 x 1m^2

= 3.850 cans

Figure 2. The Translation of Figure 1

In question number 5, TSK still had difficulty determining the solution to the problem, whether with the area of the circle or with the circumference of the circle as shown in Figure 3.

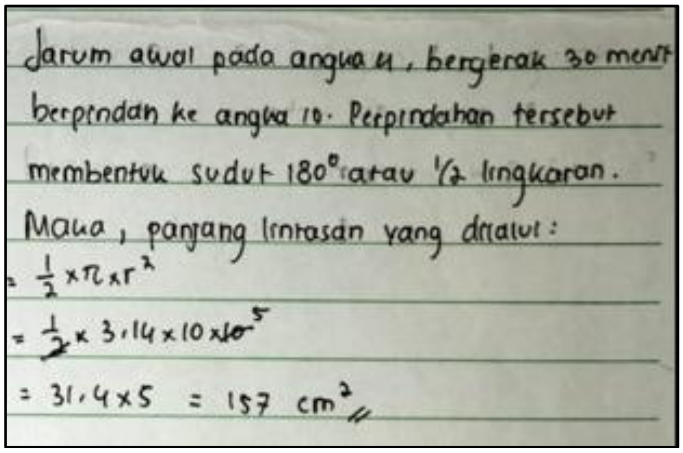

Figure 3. TSK's Answer to Question Number 5

The translation of Figure 3 is shown in Figure 4.

The initial needle at number 4, it moves 30 minutes and moves to number 10. The movement forms an angle of 180^0 or $\frac{1}{2}$ circle. So, the length of the path traversed

$$= \frac{1}{2}\pi r^2$$
$$= \frac{1}{2} \times 3{,}14 \times 10 \times 10$$
$$= 31{,}4 \times 5$$
$$= 157\ cm^2$$

Figure 4. The Translation of Figure 3

In the next stage, namely stage 3 (deduction), TSK did not answer the question because he did not know how to solve it as shown in Figure 5.

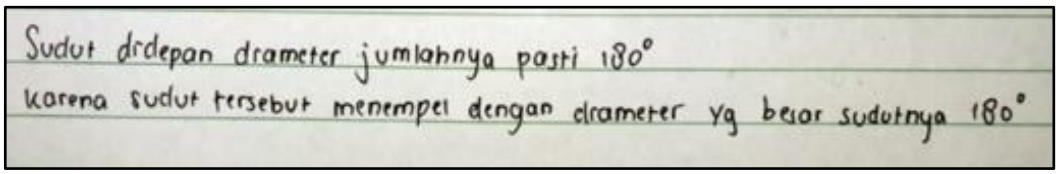

Figure 5. TSK's Answer to Question Number 7

The translation of Figure 5 is shown in Figure 6.

The angle opposite the diameter must add up to 180^0 because the angle is attached to the diameter whose angle is 180^0.

Figure 6. The Translation of Figure 6

Based on the results of student answers analysis and interview, it was found that the TSK geometric thinking ability was at stage 2 (informal deduction).

Subjects with Medium KAM (DAP)

DAP did very well on stage 0 (visualization) questions, it was just that there were some parts that were not thorough. As at the time of the test, DAP did not mention that the wheels and bottle caps were visualizations of circles. But when confirmed DAP realized his mistake. In stage 1 (analysis) most of the DAP answered correctly, but forgot about the apothem element. In addition, DAP also forgot the formula for finding the area of a square. Still at stage 1, question number 3, DAP incorrectly answered the rotational symmetry in a circle of six.

Stage 2 (informal deduction), DAP had correctly solved the problem with the formula for the area of a circle, only made a mistake in computing as shown in Figure 7.

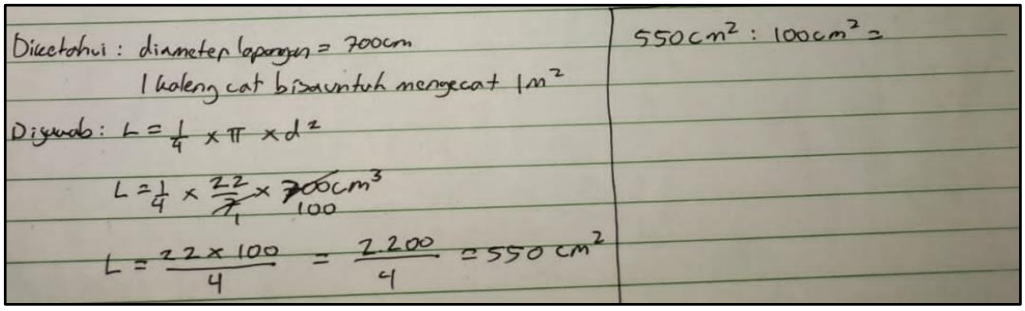

Figure 7 DAP's Answer to Question Number 4

The translation of Figure 7 is shown in Figure 8.

Known:
Diameter= 700 cm
1 can of paint can be used to paint 1 m^2
Answered:

$$L = \frac{1}{4} \times \pi \times d^2$$

$$L = \frac{1}{4} \times \frac{22}{7} \times 700\ cm^3$$

$$L = \frac{22 \times 100}{4} = \frac{2.200}{4} = 550\ cm^2$$

$550\ cm^2 : 100\ cm^2 =$

Figure 8. The Translation of Figure 7

In stage 3 (deduction), DAP did not answer the question at all, because DAP did not understand the concept of central angle and perimeter angle. The following is the reason why TSK did not answer question.

P : Why were the last two questions not answered?
DAP : I have no idea what to answer
P : Let me ask. Which one is central angle?
DAP : I don't know mom
P : What about circumferential angle?
DAP : I don't know mom

Based on the results of student answers analysis and interview, it was found that the DAP geometric thinking ability was at stage 2 (informal deduction). In line

with van de Walle's opinion stating that most students junior high school are between stages 0 (visualization) to stage 2 (formal deduction) (Van de Walle, 2001).

Subjects with Low KAM (STA)

At stage 0 (visualization) the STA made a mistake, the ball and marble were a circle. When confirmed the reason was because of its circle shape. In stage 1 (analysis), the STA could not show the apothems and gaps. STA also did not understand the relationship between the elements of a circle, such as whether the diameter and length of the chord are the same. STA was also wrong in identifying the properties of the circle. After being confirmed through interviews, it turned out that STA did not know what folding symmetry was.

In stage 2 (informal deduction), STA did not know what concept to use to solve problem number 4. STA solved problem number 4 with the concept of circumference of a circle which should be solved by the area of a circle as shown in Figure 9.

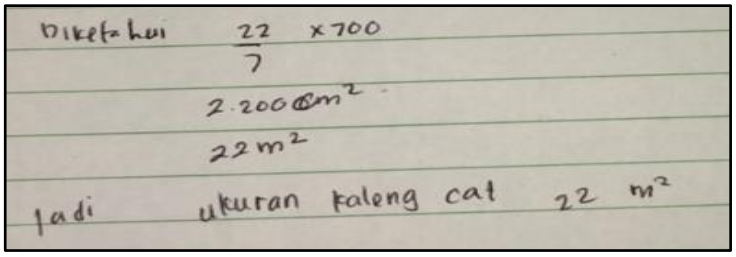

Figure 9. STA's Answer to Question Number 4

The translation of Figure 9 is shown in Figure 10.

Known: $\frac{22}{7} \times 700$

$2.200\ cm^2$

$22\ m^2$

So, the size of a paint can $22\ m^2$

Figure 10. The Translation of Figure 9

STA did not understand the questions well. The following is the researcher's interview with STA.

P	:	*What is asked in the question?*
STA	:	*Number of paint cans needed*
P	:	*How to find it?*
STA	:	*(silence)*
P	:	*What formula is this?* $\frac{22}{7} \times 700$
STA	:	$\pi \times diameter$
P	:	*What formula is that?*
STA	:	*Circumference*
P	:	*If this is the question, the whole field will be painted means finding the perimeter or area?*
STA	:	*(silence)*
P	:	*Look! Tthis is a circle yes? (draw a circle). Which circle is it? What is the area of the circle*
STA	:	*(quiet thinking)*
P	:	*The circumference is this one (thickening the curved side of the circle). If the area is this one (shading the inside of the circle). If you ask the whole field, you are asking for the area of the circle. What is the formula for the area of a circle?*
STA	:	$\pi \times r^2$

In Stage 3 (deduction), TSA did not answer the question at all, because TSA did not know at all how to prove it. Based on the results of student answers analysis and interview, it was found that the STA geometric thinking ability was at stage 0 (visualization). Subjects with a geometric thinking level of 0 (visualization), have an understanding of the concept quite good, but still hesitant in bringing up skills applied (Muarifah, 2016).

CONCLUSION

Student with high initial mathematical abilitiy, geometric thinking ability is already in stage 2 (informal deduction). Student with high initial mathematical ability has mastered the relationship between the components of the circle but has not answered the questions correctly. Student with medium mathematical ability, geometric thinking ability is at stage 2 (informal deduction). Student with medium initial mathematical ability has understood the relationship between the components

of the circle but not careful in terms of computing (counting). Students with low initial mathematical ability, geometric thinking ability is at stage 0 (visualization). Students has not been able to distinguish between flat and spaced shapes, student has not been able to explain that a circle is a flat shape.

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