



## DEVELOPMENT OF INTEGRATED DISCRETE MATHEMATICS TEACHINGBOOK WITH OPS TRANSFORMATION

Ahmad Lazwardi<sup>1</sup>, Mayada<sup>2</sup>

<sup>1,2</sup>Universitas Muhammadiyah Banjarmasin

[Lazwardiahmad@gmail.com](mailto:Lazwardiahmad@gmail.com)

### Abstrak

Transformasi ops telah terbukti secara analitis mampu menyelesaikan permasalahan matematika diskrit yang berkaitan dengan fungsi pembangkit biasa. Tujuan penelitian ini adalah mengembangkan buku ajar matematika diskrit yang sudah dimasukkan teori transformasi ops menjadi salah satu bab di dalamnya. Metode penelitian yang digunakan adalah metode penelitian dan pengembangan model 4D. Hasil validasi ahli pada buku ajar matematika diskrit dilengkapi dengan transformasi Ops menunjukkan bahwa buku sangat valid berdasarkan validasi materi oleh penilaian ahli.

**Kata kunci:** Buku Ajar; Fungsi Pembangkit; Matematika Diskrit; Transformasi Ops

### Abstract

The Ops Transformation has been analytically proven to be able to solve discrete math problems related to ordinary generating functions. The purpose of this research is to develop a discrete mathematics textbook which has included ops transformation theory as one of its chapters. The results of the expert validation of the discrete mathematics textbook equipped with the Ops transformation show that the textbook is very valid based on the validation of the material by the expert's judgement.

**Keywords:** Discrete Mathematics; Generating functions; Ops Transformation; Textbook

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## INTRODUCTION

Discrete mathematics is a branch of mathematics that specifically deals with discrete objects (Kusumaningrum and Lestari 2019). A discrete object is an object that corresponds to a subset of the set of natural numbers. Discrete mathematics is mathematics that is very useful in various aspects of life (Aminah 2018). Discrete mathematics is a branch of mathematics that has many applications



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in science and technology, both in the field of mathematics itself and in other fields such as economics, informatics, and defense(Lazwardi 2020).

Based on observations from the learning outcomes of students participating in discrete mathematics courses at UM Banjarmasin, it was found that student learning outcomes were still low. Based on interviews conducted with discrete mathematics teaching lecturers at the Mathematics Education Study Program STKIP PGRI Banjarmasin, discrete mathematics teaching lecturers at the Mathematics Education Study Program UIN Antasari Banjarmasin, information was obtained that students taking their courses still had many difficulties in learning discrete mathematics, especially in studying and solving problems -questions related to ordinary generating functions.

Based on the results of the same interview, information was also obtained that the books they used had not been updated. All the books that they use in the chapter on ordinary generating functions still use the old method, namely using the modeling of the generating function in the form of a power series (Wilf 2018).

From the results of previous studies, a new calculation method was found to solve ordinary generating function cases using the ops transformation. The ops transformation is defined as follows:

$$Ops(\{a_n\})(x) = \sum_{n=0}^{\infty} a_n x^n.$$

Lazwardi have mentioned in his paper about thre properties of Ops Transformation (Lazwardi 2021):

**Theorem 1.** For each  $\{a_n\} \in \mathbb{R}^{\mathbb{N}}$ ,  $Ops(\{0, a_0, a_1, \dots\}) = xOps(\{a_n\})$

Proof: For each  $x$  we have

$$xOps(\{a_n\})(x) = x \sum_{n=0}^{\infty} a_n x^n$$



$$\begin{aligned}
 &= a_0x + a_1x^2 + \dots \\
 &= 0 + a_0x + a_1x^2 + \dots \\
 &= Ops(\{0, a_0, a_1, \dots\})
 \end{aligned}$$

Inductively theorem 1 gives consequence as follow

**Corollary 2.** For each  $\{a_n\} \in \mathbb{R}^{\mathbb{N}}$ ,  $Ops\left(\underbrace{\{0, 0, \dots, 0\}}_{k\text{-terms}}, a_0, a_1, \dots\right) = x^k Ops(\{a_n\})$

From above theorem, we have following properties

**Theorem 3.** For each  $\{a_n\} \in \mathbb{R}^{\mathbb{N}}$ ,  $Ops(\{a_n\}) - a_0 = xOps(\{a_{n+1}\})$ .

Proof: For each  $x$  lies on the domain we have

$$\begin{aligned}
 Ops(\{a_n\})(x) - a_0 &= \sum_{n=1}^{\infty} a_n x^n \\
 &= \sum_{n=1}^{\infty} a_{n+1} x^{n+1} \\
 &= x \sum_{n=1}^{\infty} a_{n+1} x^n \\
 &= xOps(\{a_{n+1}\})(x).
 \end{aligned}$$

Thus inductively we get

**Corollary 4.** For each  $\{a_n\} \in \mathbb{R}^{\mathbb{N}}$ ,  $Ops(\{a_n\}) - \sum_{n=0}^{k-1} a_n x^n = x^k Ops(\{a_{n+k}\})$

As another consequence we also have

**Corollary 5.** For each  $\{a_n\} \in \mathbb{R}^{\mathbb{N}}$ ,  $Ops(\{a_{n+k}\}) = \frac{Ops(\{a_n\}) - \sum_{n=0}^{k-1} a_n x^n}{x^k}$ ,  $x \neq 0$



Ops transformation is failed to be linear transformation from  $\mathbb{R}^{\mathbb{N}} \times \mathbb{R}$  to  $\mathbb{R}^*$  because  $Ops(\{a_n\})(x + y)$  is not necessary equals to  $Ops(\{a_n\})(x) + Ops(\{a_n\})(y)$ . But if we set  $x$  to be fixed then we have

$$Ops(\{a_n\} + \{b_n\})(x) = Ops(\{a_n\})(x) + Ops(\{b_n\})(x) \quad (1)$$

By ignoring  $x$  for a while we can write

$$Ops(\{a_n\} + \{b_n\}) = Ops(\{a_n\})(x) + Ops(\{b_n\}) \quad (2)$$

Its easy to check that for each  $\alpha \in \mathbb{R}$

$$Ops(\alpha\{a_n\}) = \alpha Ops(\{a_n\}) \quad (3)$$

This Ops transformation is scientifically proven to be able to solve ordinary generating function cases (Lazwardi, Ariyanti, and Djamilah 2022). This research is certainly a development of previous research. The purpose of this research is to develop a discrete mathematics textbook that incorporates the Ops transformation theory as a method for solving problems related to discrete mathematics.

## METHOD

This research is development research. The validation subjects used for the development of an integrated discrete mathematics learning book with the Ops transformation consisted of two mathematicians who were experts from the field of pure mathematics and from the field of mathematics education. The process of collecting data from this research is to use a validated expert validation questionnaire and the effectiveness has not yet been tested because it is not yet in the limited trial stage.

The method used is the 4D method. First stage (define): At this stage, the products to be developed and product specifications are defined, including the



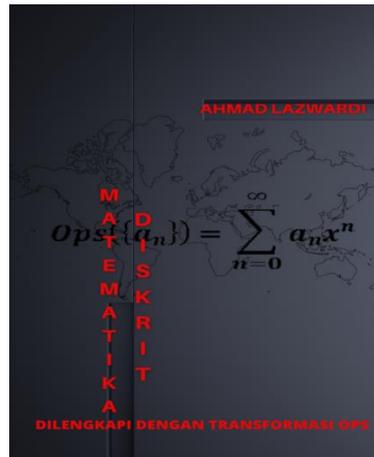
identification of problems that occur in the public (Harahap et al. 2023). Second Stage (design): At this stage the book to be developed is designed. In this stage expert validation is also carried out (Lazulfa and Putra 2020). The validators were chosen by 2 experts. One expert from the field of Pure Mathematics (major in Mathematical Analysis and is an administrator in the scientific association KAMINDO). One expert from the field of Mathematics Education (selected from the national association of mathematics education I-MES who has taught discrete mathematics at accredited A universities).

Third Stage (develop): At this stage improvements are made based on reviews from validators (Nurmeidina, Lazwardi, and Nugroho 2021). At this stage, layout improvements are also carried out. Fourth Stage (disiminate): At this stage product socialization and field trials are carried out (Oktaviana and Susiaty 2020).

## RESULT AND DISCUSSION

At the define stage, observations were made in the form of interviews with discrete mathematics teaching lecturers from the UM Banjarmasin Information Technique (IT) Study Program, Mathematics Education Study Program UIN Antasari Banjarmasin, Mathematics Education Study Program STKIP PGRI Banjarmasin. All of the respondent were confessing that they found many difficulties to teach Discrete Mathematics expecially on chapter Genetating Functions, Recurrence relations, and Derangements which involving some more complicated theory of power series manipulations. Based on the results of the interviews, then a plan was made to make a textbook with characteristics as following specifications. The next stage is design. At this stage a discrete mathematics book is designed with specifications:





**Figure 1: Book Cover**

To carry out an assessment from an expert, an assessment questionnaire was also designed which was then validated by the expert before the questionnaire was filled in by the discrete mathematics textbook validator. The questionnaire contains 6 aspects with 29 questions. The points are: A) Material and concept description aspects include the truth of the concept is based on standards used on an international scale, the correctness of the concept is based on standards used on a national scale, appropriateness of the order of the material, the accuracy of the material according to discrete mathematics branches (discussion of the material does not extend to other branches of mathematics such as calculus and real number algebra except as examples and application of theory), and material equipment. B) Compatibility with the curriculum includes compatibility with the discrete mathematics course (syllabus/RPS) presented in the study program where I teach, the book is in accordance with the syllabus for discrete mathematics courses in accordance with Course Learning Outcomes (CPMK) for study program standards nationally, and the material in the book is too advanced (too high does not match the S1 level in my study program). C) Example questions, practice questions and final test of the material includes the relationship between the example and the material, correspondence of the material with the title of the book, the contents of the book do not violate scientific, linguistic, and social ethical codes, then the exercises presented are able to measure how much learning achievement is imposed



on my course, the practice questions are sufficient to measure student abilities, practice questions representing Lower Order Thinking (LOTS) and Higher Order Thinking (HOTS) types, and the answer key is correct. D) Specific questions regarding Transformation Ops including the concept of the Ops transformation is correct, in my opinion, the Ops transformation material is easy to understand, Ops transformation material is easy to teach students, then the Ops transformation material is a fitting addition as a signature feature of this book and the Ops Transformation material is relevant and applicable to my area of expertise. E) Language, appearance and quality of the book include in my opinion, the language and terms used are appropriate and easy for students to understand, the math symbols used are just right, the thickness of the book is correct, the layout of the book is appropriate, this book is relevant for today's use, and this book is likely to still be relevant for the next 5 years. F) Book quality and marketability include paying attention to its quality, this book is worth selling to students, by paying attention to its quality, this book is suitable for sale to the general public, and by paying attention to its quality, this book is worth selling based on the standard price of math books that apply in general.

The question instruments are decided by making some consultation with the instrument validator, and taking range between 1-5. Scoring and making conclusion will be referred to Likert Scale as following (Pranatawijaya et al. 2019):

**Table 1. Scoring Guideline**

	Aspects Score	
	<i>Conclusion</i>	<i>Recommendation</i>
$0 \leq x < 1$	Not relevant	The product cannot be used
$1 \leq x < 2$	Almost not relevant	The product should have major repair.
$2 \leq x < 3$	Quite relevant	The product should have to be repaired before used.
$3 \leq x < 4$	Relevant	The product is recommendedly have a little repair
$4 \leq x < 5$	Very relevant	The Product is fine to be used



Next activity is the develop or development stage. After designing the book, this book was then validated by experts from Pure Mathematics and Mathematics Education. The results of the assessment of the validator and the aspects assessed are presented in the following table (Sari 2023):

**Table 2. Validation Results**

	Aspects Score					
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
Validator 1	4,6	4,3	4,28	4,4	4,8	5
Validator 2	4,2	3,67	4,14	3,6	4	4
Average	4,4	3,98	4,21	4	4,4	4,5
Overall Score	4,25					

On the *A* aspect, there is no significant difference score between validator 1 and validator 2. Both of them have good point view of this aspect which can be seen from the scores given by these two validators.. On the *B* aspect, there is a significant difference in the values of the two validators. Validator 1 has a good view in this aspect. Unlike the case with validator 2 assumes that the content doesn't too compatible with the discrete mathematics course (syllabus/RPS) presented in the study program where validator 2 teach. It's natural because in this book, the author tries to combine the syllabus generally from 3 different study program and make priority for the common content.

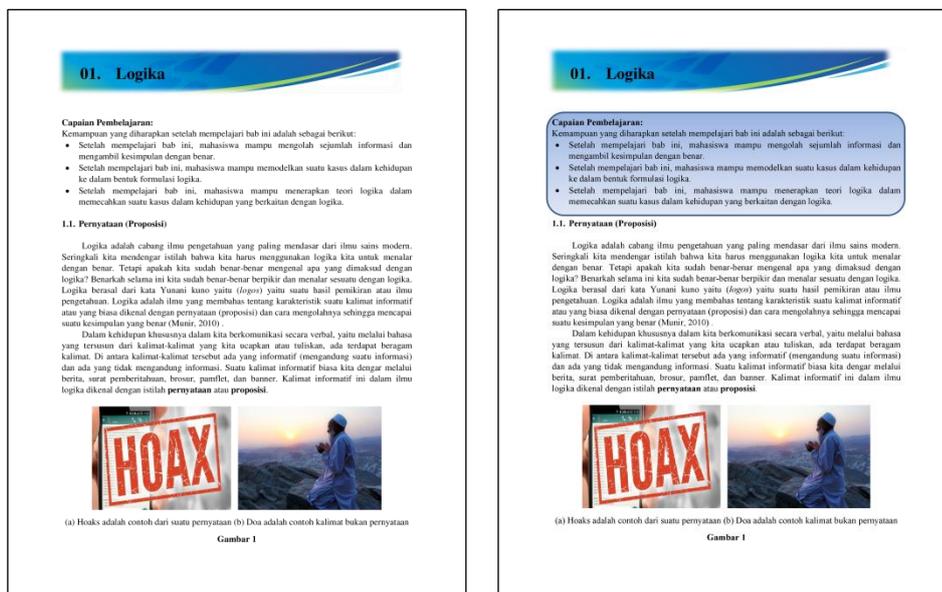
On the *C* aspect, there is no significant difference. Either from validator 1 or validator 2 both give almost the same value which indicates they have more or less the same views on this aspect. On the *D* aspect, validator 1 gives a good score but in contrast to validator 2, it seems to have doubt about the Ops Transformation will become easy to be taught and understood for commonly students in his study program. This is will become next plan to research more detail and scientifically.

On the *E* aspect, there is no major problem to be discussed. This is because it can be seen from the value given, both from validator 1 and validator 2 so that it indicates that in this aspect both have been well regarded. On the *F* aspect, validator



1 seems to be very confident that the book will be easy to be sold and will become greatly marketable. Overall score is 4,25 indicates tha the book is already fine to be used.

Beside giving the score, validators give many suggestions for revision of the textbook such as layout aspects, explanation should be written more briefly, and consistency of figures and symbols. In layout aspects, learning outcome should be given the background as the highlight. The revision could be view as following.



Before

After

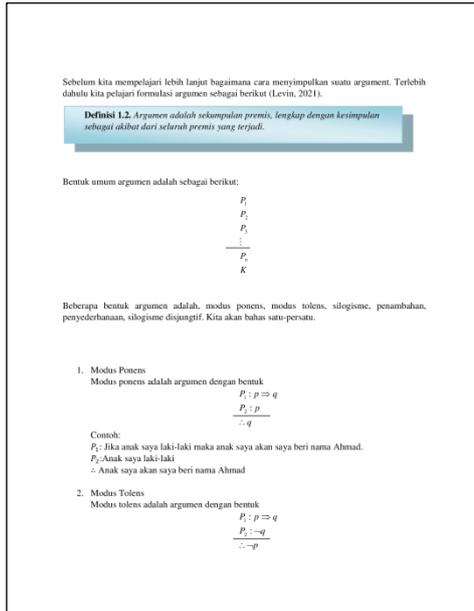
Figure 2: Layout Revision

Another recommendation from validator is about explanation. Some explanation is still less and need to be added, such as following.

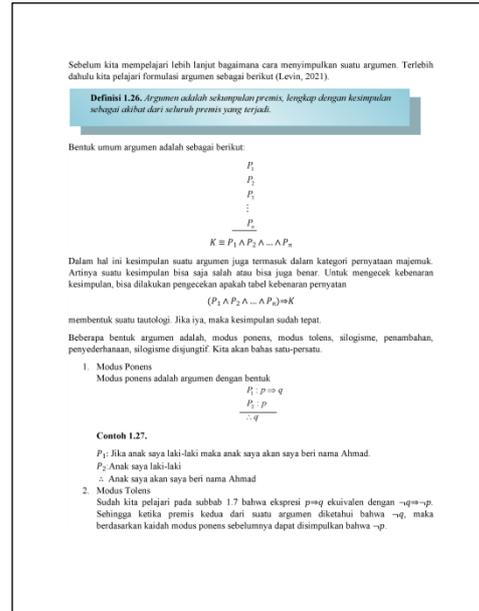


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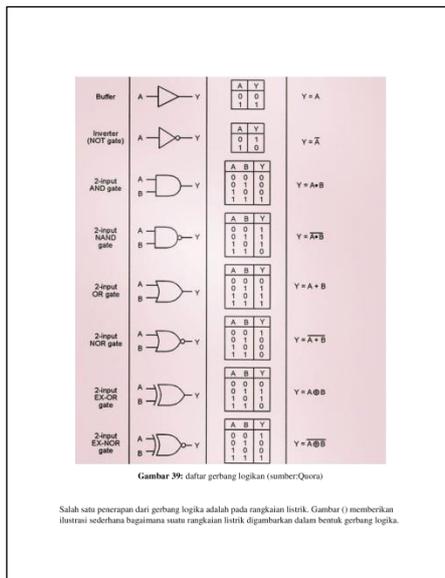
Before



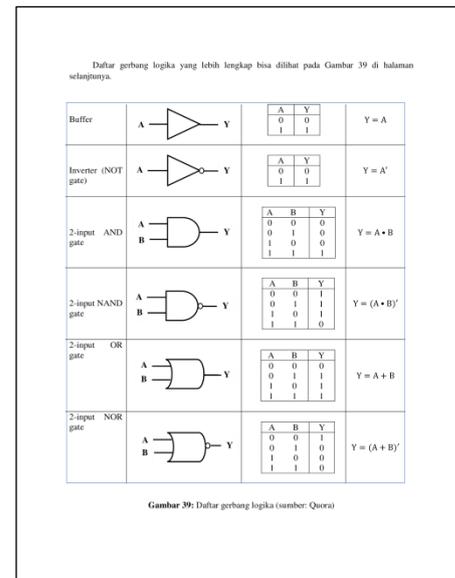
After

Figure 3: Explanation Revision

Some figures and symbols are also need to be more consistent to avoid readers from any confusion and hard to understand the book.



Before



After

Figure 4: Figure Revision



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## CONCLUSION

After all, the conclusion is the book entitled “Matematika Diskrit dilengkapi dengan Transformasi Ops” is fine to be used. The novelty is a discrete mathematics book integrated with the Ops transformation which, based on this research, passed expert validation. Some recommendation for the next research is, to continue experiment research about effectiveness and practicality of this book when applied to the real lecturing activity on some class of Discrete Mathematics. The dissemination process will be continued after the next research is success.

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