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INCREASING STUDENTS' MATHEMATICAL PROOF ABILITY THROUGH COMMOGNITIVE FRAMEWORK BASED ASSESSMENT INSTRUMENT

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Abstrak

Pembuktian matematika merupakan salah satu hal yang sangat penting untuk dikuasai mahasiswa Program Studi Pendidikan Matematika. Matematika tidak akan dapat dipelajari tanpa belajar bukti dan bagaimana merekonstruksi bukti tersebut. Meskipun demikian, mahasiswa masih mengalami kesulitan ketika menemukan soal-soal pembuktian matematika di beberapa mata kuliah yang mereka programkan, salah satunya pada mata kuliah Analisis Kompleks. Peneliti menaruh perhatian khusus pada hal tersebut dan menyusun instrumen penilaian berbasis *commognitive framework* guna meningkatkan kemampuan pembuktian matematika mahasiswa. *Commognitive framework* terdiri atas 4 indikator: penggunaan kata, mediator visual, naratif, dan rutinitas. Instrumen yang disusun digunakan dalam penelitian tindakan kelas yang dilakukan dalam 2 siklus. Model penelitian tindakan yang digunakan adalah model DDAER (*diagnosis, design, action and observation, evaluation, reflection*). Kelas yang menjadi subjek penelitian adalah kelas 2021 B Prodi Pendidikan Matematika Universitas Sulawesi Barat yang memprogramkan mata kuliah Analisis Kompleks sebanyak 26 mahasiswa. Hasil penelitian menunjukkan bahwa terjadi peningkatan kemampuan pembuktian matematika mahasiswa dengan menggunakan instrumen penilaian berbasis commognitive framework. Pada siklus I, sebanyak 39% mahasiswa memperoleh skor minimal 70, sedangkan pada siklus II, sebanyak 73% mahasiswa memperoleh skor minimal 70.
Kata Kunci: *Commognitive Framework*; Instrumen Asesmen; Pembuktian Matematika

Abstract

Mathematical proof is one of the most important things for students to master in the Mathematics Education Study Program. Mathematics cannot be learned without learning evidence and how to reconstruct that evidence. Even so, students still experience difficulties when they find problems proving mathematics in a number of courses they are programmed for, one of which is in the Complex Analysis course. Researchers pay special attention to this and develop assessment instruments based on a commognitive framework to improve students' mathematical proving abilities. The cognitive framework consists of 4 indicators: word use, visual mediators, narrative, and routine. The instruments prepared were used in classroom action research which was carried out in 2 cycles. The action research model used is the DDAER model (diagnosis, design, action and observation, evaluation, reflection). The class that was the subject of the research was class 2021 B of the Mathematics Education Study Program at the Universitas Sulawesi Barat which programmed the Complex Analysis course as many as 26 students. The results showed that there was an increase in students' mathematical proving abilities

by using a commognitive framework-based assessment instrument. In cycle I, 39% of students got a minimum score of 70, while in cycle II, 73% of students got a minimum score of 70.
Keywords: Assessment Instrument; Commognitive Framework; Mathematical Proof

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INTRODUCTION

Mathematics cannot be studied without learning evidence and how to reconstruct that evidence. In order to develop and express insights into various phenomena, mathematical proof is one powerful way. Therefore, mathematical proof skills are a formal way to express various facts or information in reasoning and justification (Hodiyanto & Susiaty, 2018). In another article, Pelc (2014) and Solow (2014) state that to justify a theorem, mathematical proof is needed which functions as arguments. Proof is the use of logical stages from what is known, such as axioms or other principles that have been proven previously, to produce a valid deductive argument so that a conclusion is obtained based on the rules of inference (Syafri, 2017). Basically, proof is a series of deductions from assumptions in the form of premises or axioms as well as existing mathematical results in the form of lemmas and theorems to produce important things from mathematical problems. Proving mathematics is a process as well as a final phase (Khoiriah, 2017). Hernadi (2008) stated that at the initial stage, understanding mathematical proof is not interesting because it is more related to symbols and logical statements rather than numbers which are usually considered characteristic of mathematics. So far, facts in mathematics have been simply believed without thinking about suspicions about the truth of these facts, without trying to prove them for themselves, even though these facts are very simple things.

At the university level, mathematical proof skills are a must for students majoring in Mathematics. Several courses, one of which is Complex Analysis, require good proof skills, considering that the problems given in these courses are related to proof. Ironically, students' mathematical proof skills, especially in the Complex Analysis course in the Mathematics Education Study Program at the Universitas Sulawesi Barat, are very low. The Complex Analysis course scores for the 2022/2023 academic year, which consist of evidentiary questions, provide results as in Table 1 below.

Table 1. The Complex Analysis Course Scores for the 2022/2023 Academic Year

Score	Number of students
A-	1
B+	7
B	7
B-	3
C	5
Total	23

As a form of "problem" in mathematics, the ability to prove mathematics requires a point of view to be improved, considering how important this ability is to have. One of the studies that has received attention in recent years and is associated with solving mathematical problems is the commognitive framework. According to Presmeg (2016), the term "commognitive" is a combination of the words "communication" and "cognitive". Commognitive analysis is used to observe problem-solving abilities, not only to see the final results, but with the "words used, "routines performed", "visual narratives" and "mediators used" used in problem solving. Commognitive emphasizes individual communication and thinking (Rossyidha, 2021). Commognitive consists of thought processes expressed in written and oral form (Setyowati et al, 2022). Kim et al. (2017) also explained commognitive as a theory about interpersonal communication relationships and cognition processes. In addition, Emanuel and Anam (2022) argue that commognitive is the result of a thought process which is communication with oneself in verbal and symbolic form. The commognitive component indicators used in mathematical proof are modified from the opinion of Zayyadi et al. (2019) and presented in Table 2 below.

Table 2. The Commognitive Component Indicators

Commognitive Component	Indicators
Word use	Write down what will be proven and what will be shown from the theorem that is the problem.
Mediator visual	Using mathematical symbols (including equations, graphs, diagrams, symbols, images, etc.) that are used for the problem solving process in the proof stage.
Narrative	Provide reasons/arguments from each stage of proof (definitions/axios/other theorems) used.
Routine	Explain the steps taken in the proof process from start to finish and draw up conclusions.

In order to improve students' mathematical skills, there are several aspects that can be manipulated, one of which is the assessment instrument. For example, research by Yuniarti & Sari (2022) which developed a mathematical problem-solving assessment instrument. Research related to the development of assessment instruments was also carried out by Fuadia, et al (2023) by conducting formative assessment analysis in improving mathematical problem-solving abilities.

Proof ability is one of the mathematical abilities, so researchers intend to develop a cognitive framework-based assessment instrument to improve students' mathematical proof abilities. The commognitive indicators described previously will be integrated into the assessment instruments created so that students will be familiar with evidentiary questions. Using this assumption, the action hypothesis in this research is that a cognitive framework-based assessment instrument can improve students' mathematical proof ability.

METHOD

The research was conducted in the 2021 B Complex Analysis class, Mathematics Education Study Program, Universitas Sulawesi Barat, academic year 2023/2024, consisting of 26 students. In accordance with the approach used, namely classroom action research with the DDAER model (diagnosis, design, action and observation, evaluation, reflection), the stages of this research are described as follows.

1. Diagnose the Problem

At this stage the researcher observed what components in solving the proof problem in the Complex Analysis course had not been optimal so far. Researchers analyzed students' UAS results from the previous academic year. Based on the diagnosis carried out, information was obtained that so far students have not been trained to work on proof questions because there are no special instruments that can help them to know the correct proof procedure in mathematics.

2. Action Planning

a. Data collection instrument

The cognitive framework-based assessment instrument used is confirmed to be ready and appropriate before use. The instruments that have been prepared are validated by 2 lecturers who have experience teaching Complex Analysis courses.

b. Action scenario

The scenario contains action steps carried out by the researcher. How and what documentation tools are needed are also prepared at this stage.

3. Implementation of Actions and Observations

After the instruments and scenarios are prepared, the next stage is to carry out actions. During the action process, researchers need observers to observe the process and impact of the action. Apart from being carried out to record results or impacts, observations can also discover what unique events occur during the action.

4. Data Analysis

Data obtained from both assessment instruments and observation sheets were analyzed quantitatively descriptively.

5. Evaluation and Reflection

Evaluation aims to make a decision on the sustainability of the action, whether it is worth continuing, needs improvement, or whether it is stopped or replaced with another action. A follow-up plan is decided if the results of the first cycle are not satisfactory and based on reflection, things are found that can still be improved. The second cycle was carried out following the previous stages until the research ended. The action is considered complete if 70% of students' scores reach 70, which is the lower limit for B grades that applies at the University of West Sulawesi. If the score obtained has not reached the target, it continues to the next cycle.

All activities in each research stage are presented in the flow diagram in Figure 1.

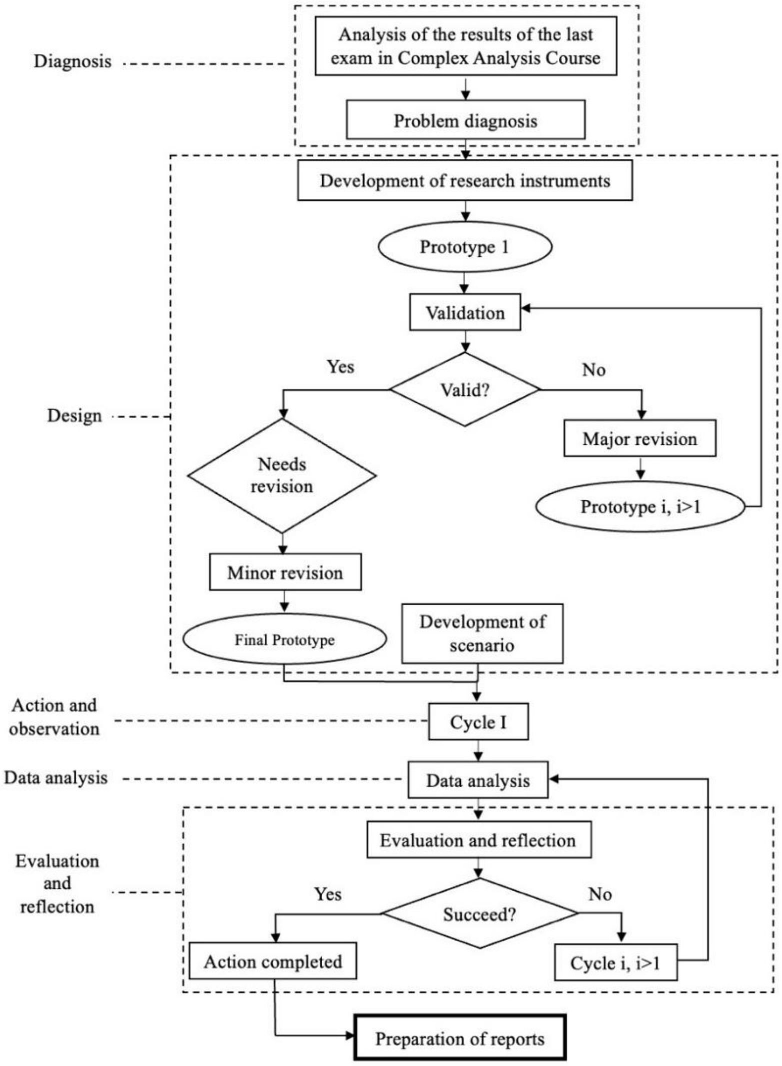

Figure 1. Flow Chart of Research Procedures

RESULT AND DISCUSSION

The research was carried out in 2 cycles on the subject matter of Complex Analysis. After carrying out the first cycle through 2 meetings, the results obtained using a commognitive framework-based assessment instrument are as shown in Table 3 below.

Table 3. The Mean Score of the Cycle I Mathematical Proof Assessment Instrument

Indicators	Meeting	
	1	2
Word use	1,3	1,2
Mediator visual	1,2	1,2
Narrative	0,9	1,0
Routine	1,1	1,2

After the 2 meetings in cycle I were carried out, then the test was held for cycle I. The test given was in the form of proving Complex Analysis material consisting of 2 questions. The results obtained from administering the test are presented in Table 4 below.

Table 4. Cycle I Mathematics Proof Test Scores

Intervals	Frequency	Percentage
$85 \leq x \leq 100$	1	3
$80 \leq x < 85$	2	8
$75 \leq x < 80$	3	12
$70 \leq x < 75$	4	15
$65 \leq x < 70$	3	12
$50 \leq x < 65$	5	19
$40 \leq x < 50$	5	19
<40	3	12

Based on Table 4, there are 10 out of 26 students who obtained a minimum score of 70. This means that there are 39% of students who meet the standard of 70 which has been determined as the standard score in this research.

Learning continued into cycle II, also for 2 meetings. The results of using a cognitive framework-based assessment instrument are as shown in Table 5 below.

Table 5. The Mean Score of the Cycle II Mathematical Proof Assessment Instrument

Indicators	Meeting	
	1	2
Word use	1,7	1,8
Mediator visual	1,9	1,7
Narrative	1,5	1,4
Routine	1,6	1,8

After 2 meetings in cycle II were carried out, then a test was held for cycle II. The test given is in the form of questions on proving Complex Analysis material which consists of 2 questions. The results obtained from administering the test are presented in Table 6 below.

Table 6 Cycle II Mathematics Proof Test Scores

Intervals	Frequency	Percentage
$85 \leq x \leq 100$	3	12
$80 \leq x < 85$	4	15
$75 \leq x < 80$	5	19
$70 \leq x < 75$	7	27
$65 \leq x < 70$	2	8
$50 \leq x < 65$	2	8
$40 \leq x < 50$	2	8
<40	1	3

Based on Table 6, there are 19 out of 26 students who obtained a minimum score of 70. This means that there are 73% of students who meet the standard of 70 which has been determined as the standard score in this research.

The research was carried out in 2 cycles, each cycle consisting of 2 meetings using a cognitive framework-based assessment instrument and 1 meeting to give the final test for each cycle. At each meeting in each cycle, a mathematical proof worksheet related to Complex Analysis material is given which is arranged according to the commognitive framework indicators.

At meeting 1 of cycle 1, students were given instructions on using the assessment instrument. When students work on questions, there are still many students who reconfirm and ask for clarification regarding their use, some students are also still confused about how to start working on these questions. At the second meeting of cycle I, students were again given instruments containing proof questions. Students are starting to work on the questions smoothly, but there are still some students who return to ask their colleagues about the working procedures. The obstacle found in cycle I was that students were not yet accustomed to using instruments, they were still focused on remembering work procedures rather than working on the questions given. Apart from that, the time given is still not enough for students to work on the questions on the instrument considering the several stages they have to write down.

Based on the obstacles found in cycle I, such as confusion regarding instructions and processing time which students considered insufficient because they had to write down the steps in full, the researcher did several things to improve this, for example explaining in more detail the meaning of each step that had to be taken. they write it down and adjust the time to complete the questions according to the difficulty level of the questions.

In cycle II, for meeting 1, students really understand the procedures for working on the instrument, so they can focus on the substance of solving the questions. Likewise with meeting 2 in cycle II, no further obstacles were found related to work instructions. The time given for working on instruments in cycle II was more than the time given in cycle I. This resulted in an increase in students' mathematics proof test scores from cycle I to cycle II.

Providing assessment instruments in cycles I and II has improved students' mathematical proof skills, especially in Complex Analysis material. By using cognitive framework-based instruments, students become accustomed to thinking and solving questions in a structured manner. Indicators of word use require students to think about what end result they want to aim for and as a first step in determining how proof will be carried out. In the visual mediator indicator, students classify what mathematical objects they want to use in solving proof problems. This facilitates the proof process that will be carried out at the next stage. Then in the narrative indicator, students write down every reason/argument they use in the proof process at each step. By doing this, students become confident in the correctness of the proof process they are carrying out. In the final indicator, namely routine, students explain the steps used as well as review and make final conclusions from the proof process. Thus, based on the research results obtained after going through 2 cycles, the action hypothesis formulated in this research can be accepted.

CONCLUSION

In cycle I, 39% of students got a minimum score of 70, while in cycle II, 73% of students got a minimum score of 70. Based on that result, it is concluded that the commognitive framework-based assessment instrument can improve students' mathematical proving abilities.

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