



## ANALYSIS OF STUDENTS' COGNITIVE STRUCTURES IN FAILED MATHEMATICAL PROOF CONSTRUCTION

<sup>\*</sup>Syukma Netti<sup>1</sup>, Suzieleez Syrene Abdul Rahim<sup>2</sup>, Listy Vermana<sup>3</sup>

<sup>1,2</sup>Bung Hatta University

<sup>3</sup>Universiti Malaya

<sup>\*</sup>Corresponding author

[syukmaneti@bunghatta.ac.id](mailto:syukmaneti@bunghatta.ac.id)

### Abstrak

Pembuktian matematis sering dianggap sulit oleh siswa, sehingga menjadi tantangan utama dalam pembelajaran matematika. Meskipun banyak penelitian telah dilakukan untuk mengeksplorasi penyebab kesulitan ini, belum ada yang menggali secara mendalam penyebab kegagalan dalam konstruksi pembuktian. Penelitian ini bertujuan untuk mengeksplorasi penyebab kegagalan siswa dalam menghasilkan pembuktian matematis yang valid melalui struktur kognitif mereka. Penelitian ini menggunakan kerangka asimilasi dan akomodasi. Metode yang digunakan adalah penelitian deskriptif kualitatif, dengan pengumpulan data melalui teknik think-aloud yang dilakukan oleh 17 mahasiswa. Setiap mahasiswa menyelesaikan soal pembuktian secara bergantian pada waktu yang berbeda, diikuti dengan wawancara untuk menggali lebih dalam struktur berpikir mereka. Dari hasil analisis data, ditemukan tiga jenis struktur skema pengetahuan yang menyebabkan kegagalan dalam pembuktian matematis, yaitu: (1) skema tidak lengkap disertai dengan skema tidak terhubung, (2) skema tidak sesuai, dan (3) skema tidak matang. Berdasarkan temuan ini, disarankan agar dosen dan mahasiswa memastikan pemahaman konsep yang mendalam, sehingga mahasiswa tidak mengalami ketidaklengkapan, ketidakterhubungan, atau ketidakmatangan dalam skema pengetahuan mereka.

**Kata kunci:** Akomodasi; Asimilasi; Konstruksi Pembuktian; Pembuktian Matematis

### Abstract

Mathematical proof problems are often considered difficult by students, making them a major challenge in mathematics education. While many studies have been conducted to explore the causes of these difficulties, none have deeply investigated the reasons behind the failure in proof construction. This study aims to explore the causes of students' failure in producing valid mathematical proof constructions through their cognitive structures. The research uses an assimilation and accommodation framework. A qualitative descriptive research method was employed, with data collected through think-aloud protocols performed by 17 students. Each student completed a proof problem alternately at different times, followed by interviews to further explore their thinking structures. Based on the data analysis, three types of cognitive structures were identified that explain the failure in mathematical proof construction: (1) incomplete cognitive structure with schema disconnection, (2) schema mismatch, and (3) immature schema. Based on these findings, it is recommended that lecturers incorporate specific learning strategies, such as concept mapping activities to reinforce conceptual



connections and structured proof validation exercises to address schema incompleteness and immaturity, thereby improving students' cognitive structures.

**Keywords:** Accommodation; Assimilation; Mathematical Proof; Proof Construction

**Citation:** Netti, S., Rahim, S.S.A., Vermana. L. 2024. Analysis of Students' Cognitive Structures in Failed Mathematical Proof Construction. *Matematika dan Pembelajaran*, 12(2), 127-142. DOI: <http://dx.doi.org/10.33477/mp.v12i2.8216>

## INTRODUCTION

Constructing mathematical proofs is a fundamental skill in advanced mathematics. However, many students face significant challenges in proof construction, as demonstrated by previous studies (e.g., Ndemo et al., 2018; Noto et al., 2019; Weber & Alcock, 2004; Maarif et al., 2019; Sears et al., 2015; Selden, 2015). One common issue is students' lack of an accurate understanding of the concept of mathematical proof (Angkotasana et al., 2024; Sangwin & Kinnear, 2021). For example, many students mistakenly believe that verifying a theorem requires only a specific example or a few cases (Erickson et al., 2021; Lee & Lee, 2016). This misconception highlights that understanding a theorem does not guarantee the ability to construct its proof.

The difficulty in constructing mathematical proofs is often attributed to a lack of **strategic knowledge**, which refers to the ability to organize and apply existing knowledge effectively during proof construction (Weber, 2001). Despite its importance, previous studies on proof construction have primarily focused on students' physical and mental behaviors (Selden et al., 2016; Selden & Selden, 1995). However, there is limited research exploring students' cognitive structures or knowledge schemas in the context of proof construction.

This study seeks to address this gap by analyzing the cognitive structures of students who fail to produce valid mathematical proofs. Building on previous research (Selden & Selden, 1995), this study aims to identify the root causes of students' difficulties, particularly their inability to unpack theorems into formal



representations. By examining students' schemas during proof construction, this research provides a more concrete understanding of the cognitive processes that underlie their struggles. The findings are expected to inform the development of targeted teaching strategies to enhance students' learning of proof material.

Thus, the primary research question is: *What are the characteristics of students' cognitive structures when they fail to produce valid mathematical proof constructions, as analyzed through the framework of assimilation and accommodation?*

Schemas, which result from the processes of assimilation and accommodation, function as tools for interpreting and organizing knowledge. Piaget's theory emphasizes that errors in the application of schemas can occur during the adaptation process (Arbib, 1990). According to Subanji & Nusantara (2016), there are five common types of errors in mathematical concept construction: (1) pseudo-construction, (2) construction gaps, (3) mis-analogical construction, (4) mis-connections, and (5) mis-logical construction. These types of errors align with Piaget's view (in Arbib, 1990) that schemas are both "intuitive" and "formal" in nature. Piaget further asserts that the acquisition of a schema does not guarantee its infallibility, and errors in schema application may lead to assimilation processes that extend the schema's applicability (Arbib, 1990: 45). From this discussion, it can be understood that the concept of a schema is relative. Schemas not only result from assimilation and accommodation, but they also serve as tools to facilitate these processes.

Cognitive development, as described by Piaget, occurs through adaptation processes involving assimilation and accommodation. Assimilation is the process of interpreting new information by integrating it into existing cognitive schemas, while accommodation requires modifications to these schemas to incorporate new experiences (Piaget in Kaasila et al., 2010). Subanji & Supratman (2015) elaborate that assimilation allows new stimuli to fit directly



into existing schemas, whereas accommodation involves restructuring existing schemas to understand and integrate the stimuli.

Building on this framework, this study analyzes students' cognitive schemas during failed mathematical proof constructions. It focuses on how incomplete assimilation and accommodation processes lead to schema-related errors, such as incomplete, disconnected, or immature cognitive structures. This analysis provides a deeper understanding of the cognitive processes underlying students' difficulties in proof construction, offering insights for developing more targeted instructional strategies.

## **METHOD**

This qualitative study explores students' cognitive structures in mathematical proof construction. Participants were selected through purposive sampling based on their familiarity with proof construction (Creswell, 2015). The study involved 17 sixth-semester Mathematics Education students at the State University of Malang who had completed Calculus I, II, and Real Analysis courses.

Data were collected using four methods: (1) Think-Aloud Protocols: Students verbalized their thought processes while solving proof problems, with recordings used for analysis. (2) Observation: The researcher noted non-verbal cues, such as sighs, to guide follow-up interview questions. (3) Interviews: Follow-up interviews explored students' reasoning, focusing on assimilation and accommodation processes. (4) Documentation: Worksheets and video recordings from the think-aloud and interview sessions captured students' reasoning.

Initially, 20 students participated, but 3 were excluded for constructing valid proofs. From the remaining 17, 6 students were chosen for in-depth analysis based on the quality of their data, assessed by: (1) completeness of proof constructions, (2) clarity of think-aloud recordings, and (3) coherence in explaining their reasoning during interviews.



The researcher served as the primary instrument, using a proof problem adapted from Selden et al. (2016) as an auxiliary tool. The problem, involving a theorem on continuous functions from Calculus, was modified to examine students' cognitive structures. The developed instrument has been validated by mathematicians and mathematics education experts from the State University of Malang.

To trace students' thought processes, the researcher developed a framework for proof construction stages. This framework was adapted from Polya's problem-solving stages (1973), Mason et al.'s Tackling Questions (2010), and pre-research observations (Netti et al., 2016). The stages are presented in Table 2.1.

**Table 1. A Comparison of Theoretical Frameworks for Problem Solving**

Problem Solving (Polya, 1973)	Tackling Question (Mason et al, 2010)	Stages of thinking Constructing Mathematical Proof
Understand the problem	Entry	Understand the problem proof
Devise a plan/ find a plan	What I Know	Create connections and selecting
	What I want	Find the main idea proof
Carry out the plan/execute	What I Introduce	
	Attack (execute) and if Stuck, re-entry is carried out	Organizing ideas/execute
Look Back	Review	Reflect at every stage

Data analysis was conducted in six stages based on Creswell's (2015) framework, which was adjusted to the focus of the study on students' cognitive structures during the construction of mathematical proofs. An explanation of each stage can be seen in the following table.

**Table 2. Data Analysis Stage**

Stage	Description
Preparing data	Transcribing and segmenting think-aloud protocols, interviews, and proof documentation.
Initial Coding	Identifying schema errors and reasoning patterns based on cognitive processes.
Overview Development	Summarizing discrepancies between students' schemas and ideal proof schemas.
Findings Representation	Categorizing themes based on assimilation and accommodation framework.



Result Interpretation	Explaining schema-related errors in failed proof constructions.
Validation	Using triangulation and member checking to ensure accuracy.

## RESULT AND DISCUSSION

The results of mathematical proof construction from 20 research subjects varied. Three people succeeded in producing valid proof construction and 17 people failed to produce valid proof. The results of the cognitive schema analysis of 17 failed mathematical proof construction results obtained 3 characteristics of students' cognitive schemas, as presented in the following table.

**Tabel 3. Characteristics of Students' Cognitive Schemas**

No	Characteristics of students' cognitive schemas	Number
1.	Complete, appropriate and mature Scheme	3
2.	Incomplete Scheme accompanied by Unconnected Scheme	5
3.	Inappropriate Scheme	6
4.	Immature Scheme	6
	Total	20

One student was selected, paying attention to the requirements that had been set, from each characteristic of the cognitive schema that failed to produce valid mathematical proof to be presented in this paper. The following is a presentation of the findings and discussion:

### **1. Incomplete Scheme accompanied by Unconnected Scheme (Subject 1 - S1)**

#### ***Understand the problem proof***

S1 experienced difficulty in understanding the proof due to an incomplete schema. For instance, S1 only understood the functions  $f$  and  $g$  as simple functions without grasping their context or purpose. The schema disconnection was evident when S1 reread the problem but couldn't use the given information to initiate the proof process..

#### ***Create connections and selecting***

S1 tried to relate the problem to the definition of continuous functions, but the definition was incomplete, covering only three basic conditions. The disconnection appeared when S1 couldn't integrate the definition with other relevant properties, such as limits or the sum of functions.



### *Organizing ideas/execute*

When trying to construct a proof, S1 used the properties of limit and addition of functions separately without realizing their relationship. After discovering how to obtain the sum  $f + g$ , which is done by adding the two equations, S1 immediately drew a line below the two equations and added them together. In this case, S1 experienced assimilation and correctly initiated the steps to construct a proof, as shown in Figure 1 below.

$$\begin{array}{l} f \text{ continuous} \Rightarrow \lim_{x \rightarrow a} f = f(a) \\ g \text{ continuous} \Rightarrow \lim_{x \rightarrow a} g = g(a) \\ \hline \lim_{x \rightarrow a} f + \lim_{x \rightarrow a} g = f(a) + g(a) \end{array}$$

**Figure 1. S1 Work Result**

However, the assimilation process stopped, S1 could not continue it. S2 tried to carry out the accommodation process as evidenced by the questions raised by S1. First, S1 questioned, “Can  $f(a) + g(a)$  be rewritten as  $(f + g)(a)$ ?” This indicates that S1’s schema regarding the property of the sum of limits is incomplete. Second, S1 asked, “How can I prove that the equation  $\lim_{x \rightarrow a} (f + g) = f(a) + g(a)$  is correct?” This shows that S1’s schema is incomplete regarding the definition and properties of the sum of limits.

The following chart compares S1's thinking process with the ideal proof scheme (problem structure). S1's fragmented knowledge structure impeded the proof construction, consistent with previous research by Selden & Selden (2015) and Wibawa et al. (2017).





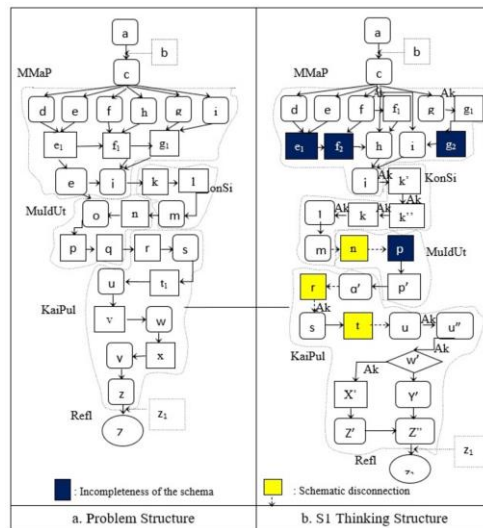


Figure 2. Comparison of Problem Structure with S1 Thinking Structure.

## 2. Inappropriate Scheme (Subject 2 - S2)

### *Understand the problem proof*

S2 misinterprets functions  $f$  and  $g$  as identity functions, reflecting a schema mismatch with the definition of functions sharing the same domain and codomain ( $A \rightarrow A$ ). This misconception leads to an incorrect understanding of continuous functions, which impacts subsequent proof steps.

### *Create connections and selecting*

S2 tries to define a continuous function, but the definition used is incorrect. S2 thinks that the function  $f$  is continuous at  $a$ , meaning the limit value of  $f$  approaches to  $a$ , as shown in Figure 3 below.

$$\lim_{u \rightarrow a} f = a$$

$$\lim_{u \rightarrow a} g = a$$

$$f + g ?$$

Figure 3. S2 Work Result

The schema mismatch is further evident when S2 applies the limit property to the sum of functions  $f$  and  $g$ , but violates mathematical principles in the process.



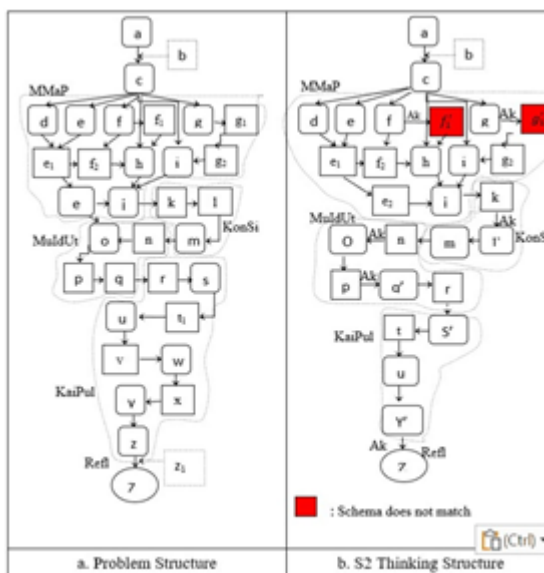


**Organizing ideas/execute**

At this stage, S2 applies the concept of direct proof and limit addition properties based on his existing, incorrect schema. Consequently, the proof construction is invalid, as illustrated by the erroneous final form:

$$(f + g) = 2a$$

This occurs despite S2 initially using a direct proof approach. Figure 4 compares S2's thought process with the ideal proof structure.



**Figure 4. Comparison of Problem Structure with S2 Thinking Structure**

**3. Immature Scheme (Subject 3 - S3)**

*Understand the problem proof*

S3 shows confidence in recognizing the proof problem, but his immature schema leads to confusion between the concepts of derivatives and continuous functions. While S3 demonstrates assimilation, his incomplete schema of the definition of a continuous function results in errors, as seen in Figure 5.

Defenition :

1.  $f(a)$  defined
2.  $\lim_{x \rightarrow a}$  defined
3.  $\lim_{x \rightarrow a} f(x) = f(a)$

**Figure 5. S3 Work Result**

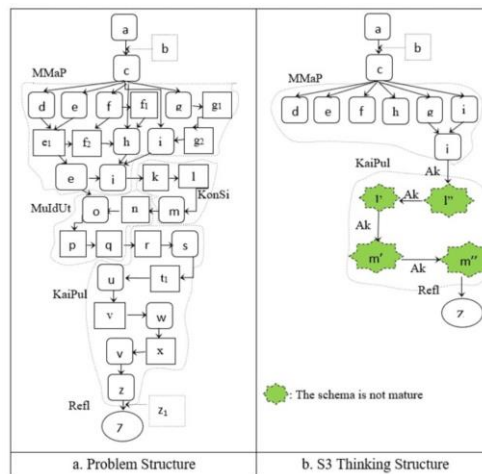


**Create connections and selecting**

S3 attempts to connect various concepts, such as the definitions of limits and epsilon-delta. However, these connections remain fragmentary and unintegrated. For instance, S3 recalls symbols like epsilon and lambda but fails to articulate the complete definition of epsilon-delta.

**Organizing ideas/execute**

Due to his immature schema, S3 struggles to logically organize the proof. He attempts to use the epsilon-delta theorem but cannot apply the symbols correctly. Repeated efforts to construct the proof fail, and S3 ultimately abandons the task. Figure 6 compares S3's thought process with the ideal proof schema.



**Figure 6. Comparison of Problem Structure with S3 Thinking Structure**

This study highlights the role of assimilation and accommodation in schema formation during proof construction. Assimilation occurs when students apply existing schemas to new problems, as seen in S2 (schema mismatch), where the identity function is mistakenly associated with functions  $f$  and  $g$ . Accommodation, on the other hand, requires modifying schemas to understand new concepts. In S1 (incomplete schema), the accommodation process fails due to insufficient existing schemas.

Students with immature schemas (S3) demonstrate early-stage, piecemeal, and unstructured cognitive development, aligning with Piaget's assertion that



schemas require reinforcement through experience (Arbib, 1990). These findings also support prior studies: Selden & Selden (1995) identified difficulties in formalizing theorems, Weber (2001) noted that a lack of strategic knowledge hinders proof construction, and Subanji & Nusantara (2017) categorized common mathematical errors such as mis-analogical construction and mis-connection.

This study contributes to the literature by: (1) identifying how incomplete, inappropriate, and immature schemas affect students' proof stages, (2) linking assimilation and accommodation processes to failure patterns in mathematical proofs, (3) offering a framework for analyzing proof failures based on thinking stages.

To address schema issues, the following strategies are proposed:  
**Incomplete/Disconnected Schemas:** Use concept mapping to help students integrate knowledge (Woldeamanue et al., 2020). Assign tasks that emphasize relationships, such as linking continuous function definitions with limit properties.  
**Inappropriate Schemas:** Apply exploratory learning to correct errors, such as comparing identity functions with functions defined in specific domains. Encourage reflection through group discussions to identify misconceptions (Susanto, 2020).  
**Immature Schemas:** Provide systematic training to strengthen basic concepts, with ongoing feedback that emphasizes gradual schema reinforcement.

Additionally, to prevent schema disconnection, integrate teaching approaches using visualizations and diagrams (Tiwari et al., 2021). Present diverse proof examples involving formal definitions to build students' conceptual connections. For immature schemas, employ gradual challenges that promote abstraction and integration, supported by problem-based learning for deeper understanding.

## CONCLUSION

This study highlights how incomplete, disconnected, inappropriate, and immature cognitive schemas influence students' failure in constructing



mathematical proofs, particularly through the processes of assimilation and accommodation. The findings indicate that: (1) Incomplete schemas accompanied by disconnected schemas (S1) cause difficulties in understanding problems and integrating concepts, contributing to failure at the initial stage of proof construction, (2) Mismatched schemas (S2) result in invalid proof steps, even when students possess the necessary basic concepts, and (3) Immature schemas (S3) lead to fragmented and unstructured thinking, causing failure to formally assemble mathematical proofs. From the 17 students analyzed, 35% exhibited incomplete and disconnected schemas (S1), 29% showed mismatched schemas (S2), and 36% demonstrated immature schemas (S3). These findings emphasize that successful proof construction requires cognitive schemas that are complete, appropriate, and mature.

This study contributes to cognitive development theory by demonstrating how schema structures and their assimilation and accommodation processes impact mathematical proof construction. The results provide a basis for developing targeted instructional strategies that strengthen students' cognitive schemas. For instance, educators can implement activities such as schema refinement exercises, structured proof validation, and concept integration practices to address specific cognitive weaknesses. This study focused on a small sample of students from a single institution, which may limit generalizability. The think-aloud protocol may also not fully capture all cognitive processes during proof construction. Future studies should involve a larger and more diverse sample to enhance the generalizability of findings. Exploring the role of external factors, such as instructional methods or cultural influences, on students' cognitive schemas could also provide valuable insights. Learning strategies that can be applied based on the findings include: First, Addressing Schema Incompleteness: To address schema incompleteness and disconnection, instructors can use concept mapping and structured concept integration activities. These strategies encourage students to connect isolated concepts into a cohesive schema by visually



organizing and connecting key ideas in the proof. Second, Enhancing Schema Fit: To address schema mismatch, educators should implement guided proof analysis exercises, in which students evaluate and compare examples of valid and invalid proofs. These activities help refine their understanding of proper proof techniques and improve their ability to apply accurate schemas in solving problems. Third, Supporting Schema Maturation: To encourage schema maturation, instructors can use step-by-step proof construction tasks, starting with simple proofs and gradually increasing their complexity. In addition, reflective practices, such as self-explanatory exercises and peer feedback, can help students structure their thinking and develop formal proof construction skills.

## REFERENCES

- Angkotasari, N., Suharna, H., Abdullah, I. H., & Dahlan, S. (2024). The Difficulty of Students' Reflective Thinking in Problems Solving of Linear Program. *International Education Studies*. <https://doi.org/10.5539/ies.v17n1p18>
- Arbib, M. A. (1990). A Piagetian perspective on mathematical construction. *Synthese*, 84(1), 43–58. <https://doi.org/10.1007/BF00485006>
- Creswell, J. W. 2015. Educational research : planning, conducting, and evaluating quantitative and qualitative research. Pearson Education, Inc., Boston
- Erickson, S. A., Erickson, S. A., Lockwood, E., & Lockwood, E. (2021). Investigating undergraduate students' proof schemes and perspectives about combinatorial proof. *The Journal of Mathematical Behavior*. <https://doi.org/10.1016/j.jmathb.2021.100868>
- Kaasila, R., Pehkonen, E., & Hellinen, A. (2010). Finnish pre-service teachers' and upper secondary students' understanding of division and reasoning strategies used. *Educational Studies in Mathematics*. <https://doi.org/10.1007/s10649-009-9213-1>
- Lee, K., & Lee, K. (2016). Students' proof schemes for mathematical proving and disproving of propositions. *The Journal of Mathematical Behavior*. <https://doi.org/10.1016/j.jmathb.2015.11.005>
- Maarif, S., Perbowo, K. S., & Kusharyadi, R. (2021). Depicting Epistemological Obstacles in Understanding the Concept of Sequence and Series. <https://doi.org/10.30738/indomath.v4i1.9339>



- Maarif, S., Perbowo, K. S., Perbowo, K. S., Noto, M. S., & Harisman, Y. (2019). Obstacles in Constructing Geometrical Proofs of Mathematics-Teacher-Students Based on Boero's Proving Model. *Journal of Physics: Conference Series*. <https://doi.org/10.1088/1742-6596/1315/1/012043>
- Mason, J., Burton, L., Stacey, K. 2010. Thinking Mathematically. Edisi kedua. Prentice Hall. Harlow.
- Moore, R. C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics*. <https://doi.org/10.1007/bf01273731>
- Ndemo, Z., Ndemo, Z., Mtetwa, D. J., Mtetwa, D. J., Zindi, F., & Zindi, F. (2018). Towards a Comprehensive Conception of Mathematical Proof. *Journal of Education and Learning (EduLearn)*. <https://doi.org/10.11591/edulearn.v12i4.9557>
- Netti, S. (2024). *Matematika Konstruktif Untuk Perguruan Tinggi*. Retrieved from <https://www.samudrabiru.co.id/?s=syukma+>
- Netti, S., Nusantara, T., Subanji, Abadyo, A., & Anwar, L. (2016). The Failure to Construct Proof Based on Assimilation and Accommodation Framework from Piaget. *International Education Studies*, 9(12), 12. <https://doi.org/10.5539/ies.v9n12p12>
- Noto, M. S., Priatna, N., Dahlan, J. A., & Setiyani, S. (2019). How good pre-service mathematics teacher in reading mathematical proof? *Journal of Physics: Conference Series*. <https://doi.org/10.1088/1742-6596/1280/4/042032>
- Polya, G. 1973. How To Solve It. Edisi kedua. Cetakan kedua. Princeton University Press ISBN 0-691-08097-6.
- Sangwin, C., & Kinnear, G. (2021). Investigating insight and rigour as separate constructs in mathematical proof. <https://doi.org/10.35542/osf.io/egks4>
- Sears, R & Chávez, O. 2015. Students of two-curriculum types Performance on a proof for congruent triangles. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.192-197
- Selden, A., Selden, J., & Benkhalti, A. (2016). Proof Frameworks--A Way to Get Started , submitted as a Tennessee Technological University Mathematics Technical Report , March 31 , 2016 ., (March), 1–20. <https://doi.org/10.13140/RG.2.1.4160.9368>
- Selden, J. (2015). A Theoretical Perspective on Proof Construction, Revised Version for the CERME9 Working Group 1 on Argumentation and, (December 2014).



Retrieved from <https://www.researchgate.net/publication/270105649>

- Selden, J., & Selden, A. (1995). Unpacking the logic of mathematical statements. *Educational Studies in Mathematics*, 29(2), 123–151. <https://doi.org/10.1007/BF01274210>
- Shimizu, Y. (2022). Relation Between Mathematical Proof Problem Solving, Math Anxiety, Self-Efficacy, Learning Engagement, and Backward Reasoning. *Journal of Education and Learning*. <https://doi.org/10.5539/jel.v11n6p62>
- Skemp, R. R. (1979). The Psychology of Learning Mathematics: Expanded American Edition. *The Mathematical Gazette*. <https://doi.org/10.2307/3026822>
- Subanji, & Nusantara, T. (2016). Thinking Process of Pseudo Construction in Mathematics Concepts. *International Education Studies*, 9(2), 17. <https://doi.org/10.5539/ies.v9n2p17>
- Subanji, R., & Supratman, A. M. (2015). The Pseudo-Covariational Reasoning Thought Processes in Constructing Graph Function of Reversible Event Dynamics Based on Assimilation and Accommodation Frameworks. <https://doi.org/10.7468/jksmed.2015.19.1.61>
- Susanto, S. (2020). Efektifitas Small Group Discussion Dengan Model Problem Based Learning Dalam Pembelajaran Di Masa Pandemi Covid-19. *Jurnal Pendidikan Modern Volume 06 Nomor 01*, 55-60.
- Tiwari, S. ., Obradovic, D. ., Rathour, L., Narayan Mishra, L. ., & Mishra, V. N. (2021). Visualization In Mathematics Teaching. *JOURNAL OF ADVANCES IN MATHEMATICS*, 20, 431–439. <https://doi.org/10.24297/jam.v20i.9136>
- Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*, 48(1), 101–119.
- Wibawa, K. A, Nusantara, T., Subanji, Parta, I. N. (2017). Fragmentation of Thinking Structure's Students to Solving the Problem of Application Definite Integral in Area. *International Education Studies*, 10(5), 48 – 60.
- Yenenesh Workneh Woldeamanuel, Y. W., Abate, N.T., & Berhane, D. E., (2020). Effectiveness of Concept Mapping Based Teaching Methods on Grade Eight Students' Conceptual Understanding of Photosynthesis at Ewket Fana Primary School, Bahir Dar, Ethiopia. *EURASIA Journal of Mathematics, Science and Technology Education*, 16(12) <https://doi.org/10.29333/ejmste/9276>





Yang, J., Yang, J., Zhan, L., Zhan, L., Wang, Y., Wang, Y., Moscovitch, M. (2016).  
Effects of learning experience on forgetting rates of item and associative memories.  
*Learning & Memory*. <https://doi.org/10.1101/lm.041210.115>



This work is licensed under a [Creative Commons Attribution-NonCommercial 4.0 International License](#).

