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A DIDACTICAL DESIGN FOR TEACHING CONTINUITY OF FUNCTION

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Abstrak
Siswa menghadapi kesulitan yang signifikan dalam memahami kontinuitas fungsi dalam kalkulus. Hal ini menimbulkan learning obstacle terhadap konsep kontinuitas fungsi. Oleh karena itu perlu dikembangkan sebuah desain didaktis untuk mengurangi learning obstacle yang terjadi. Penelitian kualitatif dalam penelitian ini adalah Didactical Design Research (DDR). Penelitian ini dilalukan di Universitas Khairun dimana partisipan terdiri dari dua kelompok. Pengumpulan dan analisis data adalah 1) Tes kontinuitas fungsi yang mana hasil tes ini akan dianalisis secara deskriptif 2) Pedoman wawancara mendalam dalam penelitian ini adalah untuk mengkonfirmasi jawaban mahasiswa 3) Desain didaktis untuk meminimalkan learning obstacle. Hasil penelitian menunjukkan learning obstacle awal yang diperoleh yaitu mahasiswa tidak dapat memilih fungsi mana yang akan digunakan untuk mencari nilai limit kiri dan kanan, mahasiswa mengetahui dengan baik 3 syarat kontinuitas fungsi, namun mahasiswa salah dalam menentukan limit kiri dan kanan. Ada pula mahasiswa yang tidak mengetahui syarat kesinambungan fungsi sehingga siswa tidak dapat menjawab masalah 2) Desain didaktis disusun berdasarkan Theory of Didactical Situation (TDS) 3) Dampak penerapan desain didaktis dapat memberikan pemahaman tentang kontinuitas fungsi dengan baik, walaupun masih ada learning obstacle yang muncul. Diperlukan penelitian lebih lanjut untuk memastikan learning obstacle yang muncul sesedikit mungkin.

Kata kunci: Desain didaktis; Didactical Design Research (DDR); Kontinuitas Fungsi

Abstract
Students face significant difficulties in understanding the continuity of functions that produces learning obstacle. Therefore it is necessary to develop a didactical design to enhance the teaching process. Qualitative research in this study is Didactical Design Research (DDR). This study was conducted in Universitas Khairun. Participants consist of two groups of students. Data collection and analysis 1) Test in which the results of this will be analyzed descriptively 2) In-depth interviews guideline in this research was to confirm students' answers 3) A didactical design which is used to minimize the obstacle. The results show that the Initial learning obstacle is the student cannot choose which function to use to find the left and right limit values, the student knows well the 3 conditions for continuity of function, however, the student incorrectly solves the problem. There are also the students who didn't know the conditions for continuity of function 2) Didactical design is compiled based on Theory of Didactical Situation (TDS) 3) The impact of implementing the

didactical design can provide an understanding of continuity of function, there are still learning obstacles that arise. Further research is needed to ensure that as few learning obstacle appear as possible.

Keywords: Continuity of Function; Didactical Design; Didactical Design Research (DDR)

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INTRODUCTION

Discussions about continuity of function will not be separated from the concept of limits. While limits allow for value evaluation and convergence determination, continuity ensures that functions are smooth *(*Munyaruhengeri et al, 2024). A function must be defined and continuous at a point for its limit to exist there (Messias & Brandemberg, 2015). Burazin et al (2024) continuity of functions is a fundamental concept in calculus, modeling real-world processes and serving as a foundation for more advanced mathematical topics such analysis courses (Das, 2019). Continuity in calculus is different from analysis courses which can be seen in textbooks. Continuity presentation in calculus textbooks often involves a balance between intuition and rigorous definitions. This balance is crucial, as differences in definitions between calculus and analysis courses can lead to contradictory outcomes and unexpected language (Shipman, 2012). Continuous functions possess valuable properties, such as being bounded on closed intervals, and can be generalized to apply to functions on subsets of R (Das, 2019). Additionally, continuity is closely related to other mathematical concepts, such as limits and differentiability (Burazin et al., 2024; Shipman, 2012). Understanding the various definitions of continuity is essential for students transitioning to higher-level mathematical thinking (Shipman, 2012).

Research indicates that students face significant difficulties in understanding the continuity of functions in calculus. Common challenges include misconceptions stemming from limited exposure to active learning approaches and reliance on rote memorization (Munyaruhengeri et al., 2024). As specific, Munyaruhengeri et al continue that to determine the continuity at the point, some students found it difficult to distinguish between the left and right-hand limits, then some were able to recognize the limit, but they had trouble connecting it to the domain of the function. Lauten et al. (1994) when students were asked to explain their concepts of continuity, students failed to distinguish between the symbolic and graphical representations of function. Vinner (1992) the majority of students' perceptions that for a function to be continuous is the same as being defined and to be discontinuous is the same as being undefined at a certain point. Duru, Köklü, and Jakubowski (2010) pre-service mathematics teachers showed trouble figuring out a function's continuity and differentiability at specific places as well as connecting limit, continuity, and differentiability in both graphical and symbolic representations. To address this issue, teaching strategies, approaches and designs are needed that can help students and even the teachers with difficulties in teaching the concept of continuity of function.

Some of the challenges and obstacles that have been mentioned, teaching continuity of functions effectively requires diverse approaches. In terms of textbook, Moreira and Campos (2023) carried out a study that aids in the professional growth of calculus teachers and mathematics educators. Moreira and Campos compare Calculus books and Real Analysis books that have two distinct stances on the concepts of limit and continuity of a real function. Purbaningrum and Sukmawati (2023) suggest using structured worksheets to address student difficulties in understanding continuity Fonseca & Henriques (2020) teaching experiment focusing on multiple dimensions of understanding, including concept meanings, representations, and problem-solving applications, can lead to improved comprehension of continuity. Introducing the concept through piecewise functions has shown promise in helping students identify conditions for continuity at a point (Morales & Mojica, 2020). Visual representation using software tools like GeoGebra can enhance understanding of complex concepts such as uniform continuity, allowing students to distinguish between uniformly continuous and nonuniformly continuous functions graphically (Dikovic, 2016). This is also in line with the study Tonra et al (2023) that GeoGebra is the software that is most often used as a reference in visualizing limit concepts. These approaches collectively emphasize the importance of multiple representations, structured learning materials, and technology integration in teaching continuity of functions, aiming to develop a deeper understanding of this fundamental mathematical concept. Although many approaches have been offered to improve teaching on functional continuity, existing literature related to didactical design to overcome students' learning obstacles is limited.

Learning obstacle occurs when students grasp ideas on a surface level without being able to apply them in more complicated contexts or connect them to other mathematical ideas. There are 3 types of learning obstacles. Those are ontogenical, epistemological, and didactical (Broussseau, 2002; Suyadi, 2019a). Ontogenical obstacles have to do with how prepared and mature students are mentally to absorb information. Didactical obstacles include the order in which the lecturer presents the content or the sequence in which students complete a series of learning exercises. This obstacle is evident in the stages and content of the material presentation. The instructional materials that the instructor has created and used during the learning process may potentially be the source of this challenge. The epistemological challenge, on the other hand, concerns the assignments that students are given and whether or not they can develop children's knowledge and thought processes beyond procedural knowledge. The reality of students' learning obstacles is an indication that there are problems with the didactical design that was implemented previously (Astriani., Mujib & Firmansyah, 2022). Therefore, it is necessary to know at the beginning what the student's obstacles are in working on problems regarding functional continuity. So that teaching can adjust obstacles so that they do not happen again or at least can be reduced. In this research, teaching that is based on obtaining information related to obstacle learning is called didactical design.

The didactical design refers to the stages of Theory Didactical Situation (TDS) by Brousseau (2002) which consist of 4 situations: Action, formulation, validation, and institutionalization. An action situation is in which students are tasked with coming up with a novel hypothesis; formulation is a situation in which students begin to play, argue, and converse with other groups in order to determine the learning objectives. In validation, the instructor takes on the role of a theorist and assesses the theorems that students have developed in action and formulation scenarios. Institutionalization is essentially a process that enables students to transform their prior knowledge into new knowledge by means of lecturers' reinforcement of truth values and the application of the acquired knowledge to solve issues.

Therefore, this research aims to answer three research questions: 1) How is the description of students' initial learning obstacles in continuity of function? 2) How is the didactical design for teaching continuity of function? 3) What is the impact of implementing the didactical design? The findings give lecturers the ability to provide focused instruction and interventions meant to lessen any learning obstacle that students could encounter.

METHOD

Research Design

Didactical Design Research (DDR) is the qualitative research method used in this study. Two paradigms—interpretive and critical—form the foundation of DDR (Suryadi, 2019a). Interpretive analysis is used to identify what obstacles students face in studying continuity. After interpretive paradigm analysis, the next step is to switch to the critical paradigm. The crucial paradigm, meantime, is to develop a didactical design and assess its effectiveness in overcoming learning obstacle based on implementation outcomes. Figure 1 provides an explanation of the three stages of the DDR research conducted in this study, which are based on these two paradigms.

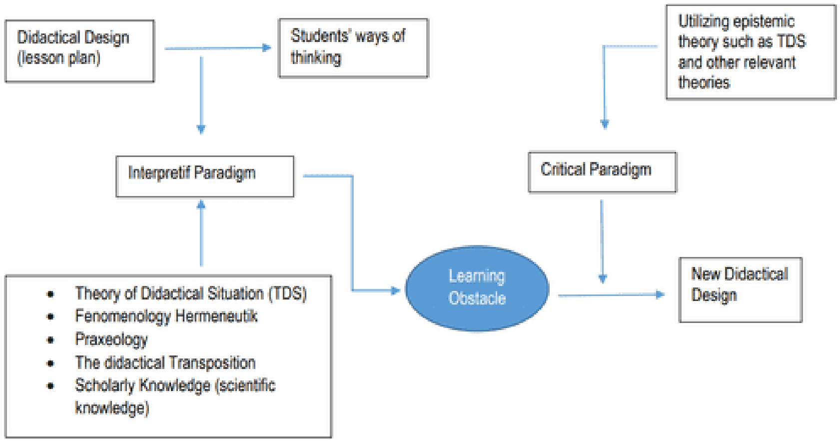


Figure 1. Interpretive and Critical Paradigm in DDR (Suryadi, 2019a)

Participants

This study was carried out at Universitas Khairun over the course of five months, from May to October 2024. There were two groups of volunteers in this study:

a. The first group (group 1) consisted of 23 students of semester 3 in mathematics education department. This group was chosen because they have passed the differential calculus course, and they were given continuity of function test to identify the initial learning obstacle.

b. The second (group 2) consisted of 33 students of semester 1 in mathematics education department. This group was chosen because the students are enrolled in Differential Calculus course. Therefore, the design that has been prepared can be applied to this group.

Data Collection and Analysis

Three instruments are used in this study to collect data. These three instruments were employed in this study.

a. The test for continuity of function

Table 1. The Test Continuity of Function

Indicators	The questions
Determine whether the indicated function is continuous at a point or discontinuous.	1. Discuss the continuity at x = 3 $f(t) = \begin{cases} t - 3, t \leq 3 \\ 3 - t, t > 3 \end{cases}$ 2. Discuss the continuity at x = 1

$$g\ (t) = \begin{cases} t+3, t \leq 1 \\ -t+1, t > 1 \end{cases}$$

b. In-depth Interview guideline

The purpose of this study's in-depth interviews was to validate the students' written answers (Turner, 2010). In order to meet research goals, the interviews' important points were condensed and then transcribed to show how students solved the problem given. Three students were interviewed in the first session to examine the learning obstacle, and three students were interviewed in the second session to observe the implementation outcomes of the didactical design. Based on the variety of responses to the function limit question—that is, the three sets of answers to the continuity function—three students were chosen. Criteria for selecting only 3 students are because there were only 3 groups of answers that appeared in the class.

c. A didactical design

This instrument was created based on: 1) the results of the initial learning obstacle analysis of students in answering continuity of function questions. 2) For the subtopics of the design, this study compares calculus books that are often used in Indonesia. There are 3 books used: 1) Varberg., Purcell, & Rigdon (Varberg, D., Purcell, E., & Rigdon, S. (2013), Koko Martono (Martono, K., 1999), and James Stewart (Stewart, J., 1999). 3) The article publication related to continuity function.

Instrument Validation Procedures

Before the test for continuity of function is given to students, the test have been validated by two specialists in the fields of mathematics education and DDR, and mathematical analysis. They provided input both in terms of content, as well as the question sentences. In this continuity test there are 2 questions that have different levels of difficulty. Meanwhile, for the didactical design, validation is used through focus group discussions (FGD). There are 5 experts who carry out validation. There are the experts in the fields of DDR, algebra, mathematics analysis, and two experts from mathematics education. In the FGD process, the instrument was presented and then 5 experts provided input both in terms of content and language.

All the stages above will represent as research process below in Figure 2.

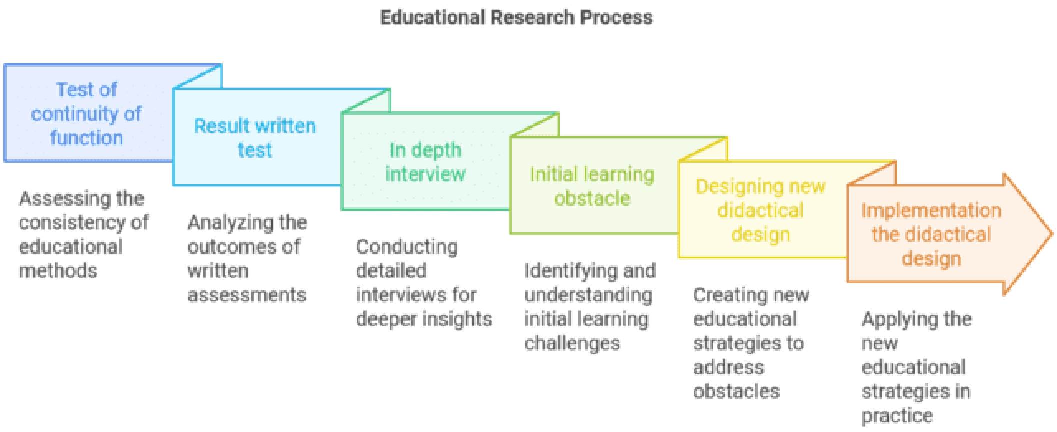


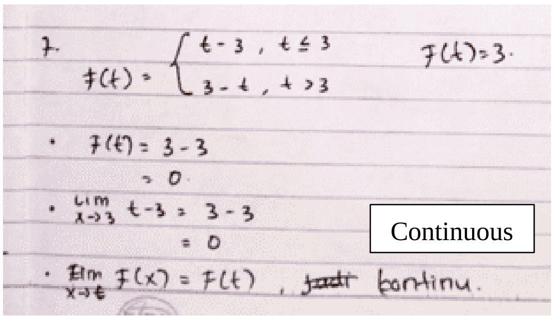
Figure. 2. Educational Research Process

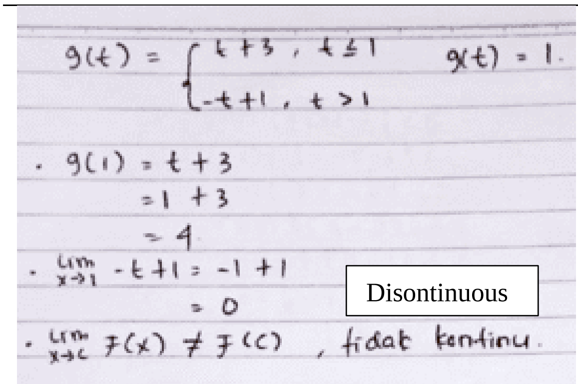
RESULT AND DISCUSSION

The Initial Learning Obstacle

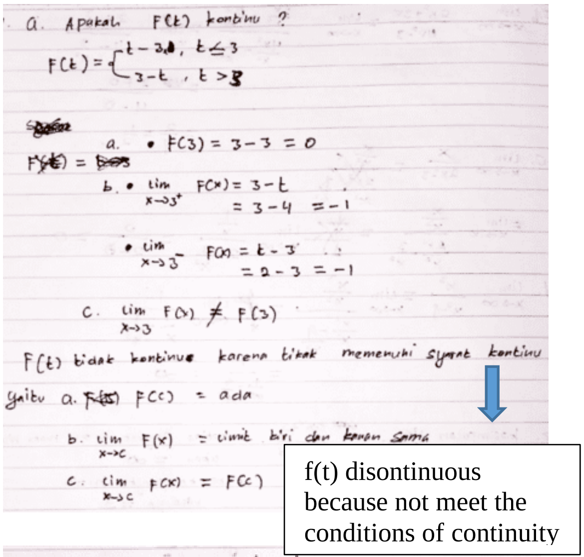
This section presents analysis of students' initial learning obstacles in understanding continuity of function, based on test results and interviews. There are 3 student answers that are representations in this study. The students are M8, M11 and M16. The following is the data presented in Table 2.

Table 2. The Initial Learning Obstacle

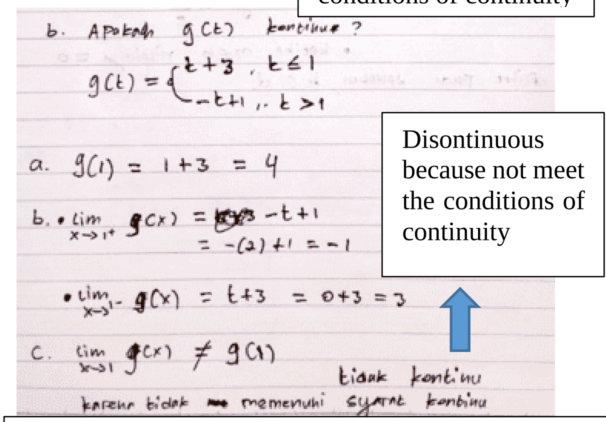
The answer	Interview conclusion
	In examining continuity conditions, M8 began by evaluating f(t) through substitution. He says to substitute the value 3 into the first function because 3 is in the first function. Then the second condition is to find the limit value, M also using the first function. The values of lim f(x) and f(t) are the same so they are continuous.



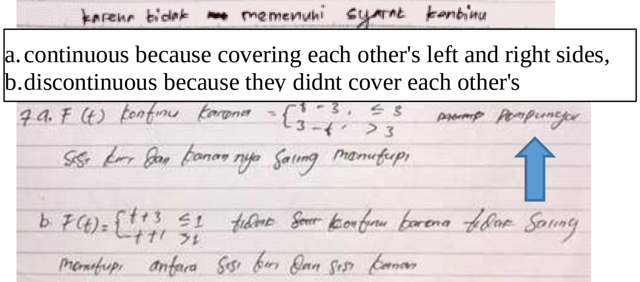
M8 looks for the value of g(t) first, he says to use the first function because 1 is in the first function. Then to find the limit value, M8 also uses the second function. The values of lim f(x) and f(t) are different so they are not continuous.



M11 starts the 3 continuity conditions by finding the value of f(3) first, then looking for the limit value. The right limit is with the second function and the left limit is with the first function. M11 says that the value of t in the right limit is 4 because M looks at the inequality sign, namely t > 3 so M11 substitutes 4. Meanwhile, for the left limit, the value of t is 2 because the inequality sign is t ≤ 3. F (t) is not continuous because the value limit is not the same as the value of f(3).



M11 starts the 3 continuity conditions by finding the value of g(1) first, then looking for the limit value. The right limit is with the second function and the left limit is with the first function. M says that the value of t in the right limit is 2 because M11 looks at the inequality sign, namely t > 1 so M11 substitutes 2. Meanwhile, for the left limit, the value of t is 0 because the inequality sign is t ≤ 1. F (t) is not continuous because the value limit is not the same as g(1).



M16 does not know how to solve this continuity problem. M16 does not know the 3 conditions for continuity of function.

Based on Table 2, M8 knows the 3 requirements for continuity of function well. However, M8 cannot choose which function to use to find the left and right limit values. M8 just determines the limit value directly without determining the left and right limits. Of course, this should not be done, even though in the end M8's answer is correct in drawing conclusions. On the other hand, M11 also knows well the 3 conditions for continuity of function. However, M11 incorrectly resolves the left and right limits as well. In the first problem, M11 got the wrong result because it took the wrong value of t. As a result, the limit value differs from the value of f (t), and in the end, it is concluded that it is not continuous. In problem number two, it is the same as in the first problem, M11 is wrong in determining the left and right limit values. For M16, he didn't know the conditions for continuity of function so he couldn't answer this question.

From learning obstacle information, the recommendation for didactical design in overcoming learning obstacles: 1) Need an explanation of how to take the appropriate function to find the left and right-hand limits. 2) Provides an overview of the 3 conditions for continuity of function from a given function or from a graphical representations.

The Didactical Design in Teaching Continuity of Function

The results of the search for published documents have never found a function limit teaching using TDS which contains 4 phases: action, formulation, validation, and institutionalization. This design also consists of 3 situations that are deliberately designed to improve students' understanding of continuity of function. Choosing TDS as a tool for didactical design because TDS can provide students with the opportunity to construct their own knowledge. Starting with action situations, where students are given situations that can stimulate students' thinking processes so that goals can be achieved. Then in the formulation stage, students are given the opportunity to draw conclusions from the actions they have taken. In the validation situation, students are given reinforcement and in the institutionalization, students apply it to various contexts. Therefore there are 2 situation in this design.

Situation 1 aims to show which graphs are continuous and which are not continuous based on 3 conditions. Situation 1 to provide students with an understanding through graphs of examples of continuous and discontinuous functions. So that students can formulate the 3 conditions for continuity of function by themselves. Situation 2 or validation situation: students are given many functions and then draw conclusions about whether the function is continuous or not continuous at a point. In Institutionalization situations, to understand the concept of continuity of function, students are given various contexts. Table 2 below is the didactical design that describe teaching activities based on TDS

Table 3. The Didactical Design of Continuity of Function

<table>
<tr><th>Teaching Activities based on TDS</th><th>Pedagogical didactic anticipation</th></tr>
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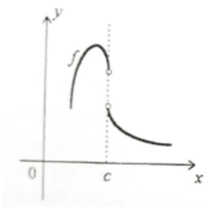
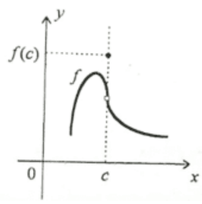
Action
Situation 1:
Given 5 types of function graphs as below:



(a)



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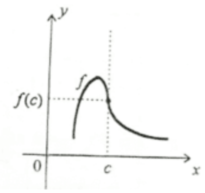

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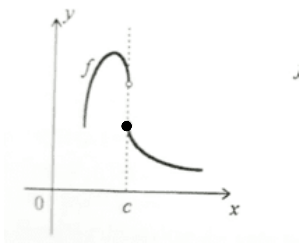
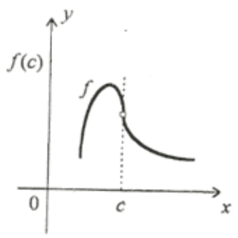


(d)



(e)

From the 5 graphs above, students are asked questions related to limit values and function values.
1) Which graph has the limit value at x = c?
2) Which graph has the function value at x = c?
3) Which graph has the limit value and the function value the same at x = c?

</td><td>

If the students cannot determine whether a graph is continuous or not :
The example of anticipation: using a geometric approach where students are asked to imagine a continuous function as a function that does not break. This graph can be drawn without lifting the pen from the paper, while a function that is not continuous or discontinuous is a function whose graph has holes, breaks, or jumps, so it requires us to lift the pen from the paper.
Other approach is remind the students about the left and right hand limits

</td></tr>
</table>

Formulation

Based on the activity in situation 1, pay attention to the graph where the limit value and function value are the same. This graph is called a graph of a continuous function. Give the conditions for the continuous function.

Validation

Situasi 2:

To understand the concept of continuity of function, students are given the following functions:

Investigate whether the function f(x) is continuous at x = 0, x =1 and x = 2

$$f(x) = \begin{cases} 2x + x^2, & x < 0 \\ \sqrt{x}, & 0 \le x < 1 \\ 2, & x = 1 \\ x^2, & 1 \le x \le 2 \\ 3 - x, & x > 2 \end{cases}$$

If the students cannot determine which function to use
The example of anticipation: use the help of a number line to illustrate the functions.

The answer:

- for x=0
  (1) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sqrt{x} = \sqrt{0} = 0$
  $\lim_{x\to 0^-} f(x) = \lim_{x\to 2^+} 2x + x^2 = 2.0 + 0^2 = 0$
  Because the left and right limits are similar then $\lim_{x\to 0} f(x) = 0$
  (2) $f(0) = \sqrt{0} = 0$
  (3) $\lim_{x\to 0} f(x) = f(0) = 0$, maka f kontinu di x= 0
- Titik x=1
  (1) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = 1^2 = 1$
  $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$
  Because the left and right limits are similar then $\lim_{x\to 1} f(x) = 1$
  (2) $f(1) = 2$.
  (3) $\lim_{x\to 1} f(x) \neq f(1)$, maka f tidak kontinu di x=1
- Titik x=2
  (1) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3 - x$ = 3-2 = 1
  $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = 2^2 = 4$

  (2) Because the left and right limits are different then $\lim_{x\to 2} f(x)$ not exist, then it is not continuous at x=2

Institutionalization

Students are given five problems in various contexts.

1.

$$f(x) = \begin{cases} 2x+1, & x \leq -1 \\ 3x, & -1 < x < 1 \\ 2x-1, & x \geq 1 \end{cases}$$

Determine whether the graph is continuous or not x=-1 dan x=1

2.

$$f(x) = \begin{cases} \frac{x^2-2x-8}{x-4}, & x \neq 4 \\ 3, & x = 4 \end{cases}$$

Determine whether the graph is continuous or not at x=4

3. Discuss whether the function is continuous or not continuous at the point x= 3
   $h(t) = \frac{|t-3|}{t-3}$
4. Investigate whether the following function is continuous or not at the x=0
   $$f(x) = \begin{cases} \frac{1}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$
5. $$f(x) = \begin{cases} 2Ax-3, & x < 1 \\ 4-x^2, & x \geq 1 \end{cases}$$

Determine the value of A so that it is continuous at x=1

The Impact of The Implementation of the Didactical Design

The efficiency of the didactic design will be examined following its implementation in group 2 learning. For instance, which learning obstacles have been solved, which learning obstacles still exist, or whether new learning obstacles are appearing (Puspita, et al, 2023) The majority of the learning challenges were determined to be effectively surmountable following the implementation of the didactical design. The results, however, show that there are still issues that need to be addressed. Even if they take on a somewhat different form, the original constraint findings remain once the didactical design is put into practice. Improvements to the didactic design will take these issues into account. The goal of this approach is to continuously raise the standard of education. Below is a detailed presentation of the analysis results following the implementation of the developed didactical design.

The Learning Obstacles that have been Solved

This is one of the works of students who have been taught using the didactical design. From this result, students can draw conclusions about whether a function is continuous or discontinuous at a point.

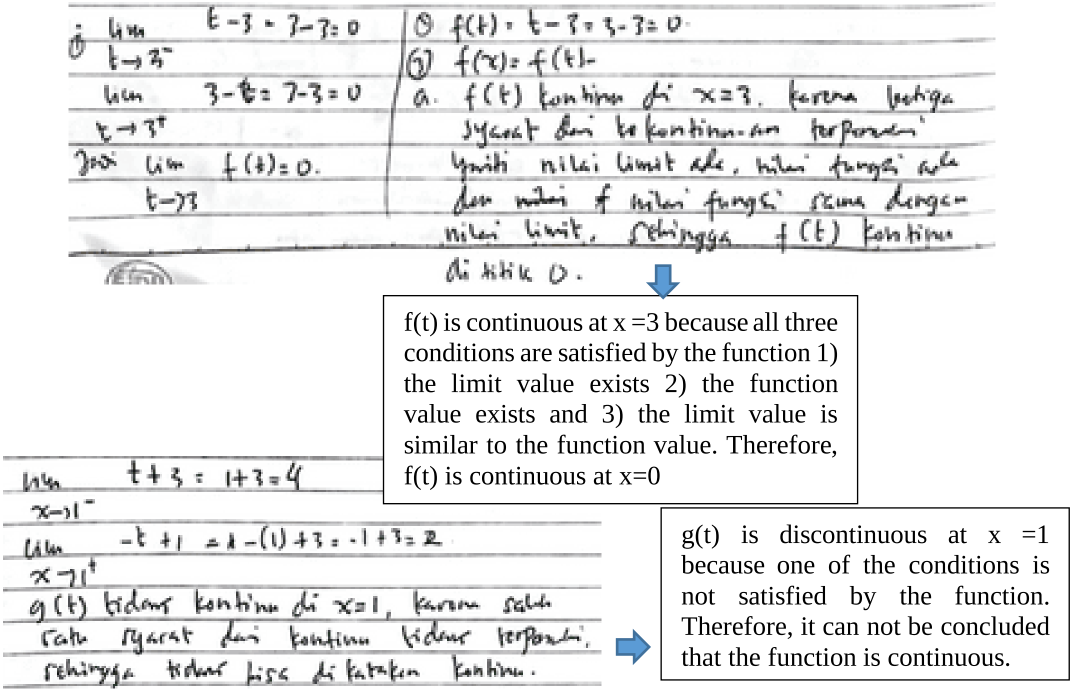
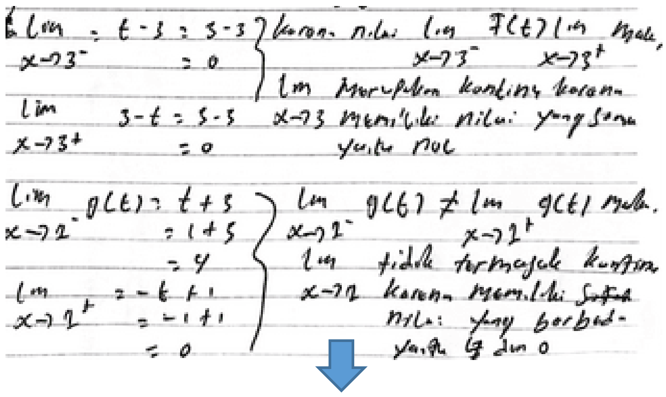


Figure. 3. The Student's Written Result of Implementing The Design

Implementation results demonstrate improved student understanding of the three continuity conditions, as evidenced by their responses to Problem 1. To sum up, the student knows to take the appropriate function to find the left and right limit limits. 2) students understand the 3 (three) conditions for continuity of function from a given function. This is in line with Tonra et al (2024) that conducted a study for single subject research, the study found that subject look the limit value with checking for the left and right limit values. Because the left and right limits are similar, the first condition of continuity is satisfied. Then the subject looks for the function value whether it is exits or not to fulfill the second condition. The subject added that the last condition is that limit of function are similar with the value of the function. Therefore the function is continuous.

a. New learning obstacles are appearing

While the didactical design resolved initial obstacles, analysis revealed emergence of new learning challenges. Based on the results of the interview and the experimental test results, students have decided that a function is continuous only by looking at the left limit and the right limit. Of course, this is an imperfect understanding, students should also check f(t) and g(t). Finally, students should check whether the limit value is the same as the value of f(t) or g(t).



Because $\lim_{x\to 3^-} f(t) = \lim_{x\to 3^+} f(t)$ therefore, the function is continuous, because both is 0

$\lim_{x\to 1^-} f(t) \neq \lim_{x\to 1} f(t)$
Therefore, the function is dicontinuous, because both is different, 4 and 0.

Figure. 4. New Obstacles are Appearing in Group 2.

The factor influencing emergence of new obstacle is that the students do not understand the 3 conditions of continuity. The student concludes that a function is continuous just by looking at the left and right limit values. The student should also check the values of f(t) and g(t) and see whether the function value is the same as the limit value.

To sum up, this study identified three key learning obstacles in understanding function continuity: difficulty in selecting appropriate functions, errors in limit calculations, and incomplete conceptual understanding. The initial learning obstacles: 1) the student cannot choose which function to use to find the left and right limit values. 2) students know well the 3 conditions for continuity of function. However, the student incorrectly solved the left and right limits as well. 3) there are also the students that didn't know the conditions for continuity of function so the students couldn't answer this question. Based on the initial learning obstacle, a didactical design is prepared. This design consists of 4 situations to reduce the initial learning obstacles. The impact of this design is that initial learning obstacles can be overcome but new learning obstacles that students faced about continuity of function are still emerging. Susilo, Darhim, and Prabawanto (2019). Their study revealed that students' critical thinking abilities relative to limits, continuity, and derivatives of functions fell into the poor range. Duru, Köklü, and Jakubowski (2010) their study's findings verified that students had trouble figuring out a function's continuity and differentiability at specific places and connecting limit, continuity, and differentiability in both symbolic and graphical representations. Munyaruhengeri et al (2024) students frequently have trouble determining if a function is continuous or discontinuous.

CONCLUSION

This study yielded three key findings regarding the teaching of function continuity. 1) The initial epistemological obstacle is the student cannot choose which function to use to find the left and right limit values, the student knows well the 3 conditions for continuity of function, however, the student incorrectly solves the left and right limits as well. There are also the students who didn't know the conditions for continuity of function so the students couldn't answer this question 2) Didactical design is compiled based on the initial learning obstacles. Phase TDS is used in teaching continuity of function. 3) The impact of implementing the didactical design is able to provide an understanding of continuity of function, although this design is not perfect because there are still obstacles that arise. The limitation of this research is that new learning obstacles still emerge and this should not happen. The reccomendation for further research is how to ensure that as few learning obstacles appear as possible so that understanding regarding continuity of function is close to perfect. The contribution of this study to mathematics education is that the results of this research provide alternative designs or methods/approaches in teaching continuity of function in calculus courses.

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