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PEER TEACHING-BASED LEARNING ASSISTED BY MANIPULATIVE MEDIA IN IMPROVING STUDENTS' UNDERSTANDING AND SKILLS IN ADVANCED MATHEMATICS COURSES

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Abstrak

Penelitian ini bertujuan untuk menganalisis pembelajaran berbasis peer teaching berbantuan media manipulatif dalam meningkatkan pemahaman dan keterampilan mahasiswa pada matakuliah Matematika lanjut. Penelitian ini menggunakan pendekatan kuantitatif deskriptif dan pendekatan kualitatif dengan melibatkan subjek penelitian sebanyak 32 mahasiswa Program Studi PGMI semester 3 Universitas Muhammadiyah Bima. Instrumen penelitian berupa tes untuk mengukur pemahaman dan keterampilan terhadap pengembangan media. Kemudian instrumen kedua berupa angket untuk mengukur persepsi mahasiswa terhadap proses pembelajaran, pengembangan media, dan pemahaman. Data dianalisis menggunakan kualitatif deskriptif untuk uraian jawaban tes, dan statistik deskriptif untuk hasil angket. Hasil penelitian menunjukkan bahwa persepsi mahasiswa terhadap proses pembelajaran peer teaching berbantuan media manipulatif berada pada kategori sangat baik (75%), pengembangan media pembelajaran berada pada kategori sangat baik (78,13%), dan tingkat pemahaman materi matematika lanjut mahasiswa berada pada kategori sangat baik (68,75%). Hasil belajar mahasiswa mencapai kriteria kelulusan dengan 100% nilai yang memenuhi standar kelulusan, dan didukung dengan keterampilan mahasiswa dalam melaksanakan peer teaching yang sangat baik. Hasil penelitian ini menyimpulakn bahwa penerapan *peer teaching* berbantuan media manipulatif mampu dapat mendorong motivasi dan persepsi positif mahasiswa terhadap proses pembelajaran matematika. Meningkatkan kreativitas dalam pengembangan media konkrit yang relevan dan interaktif. Mengoptimalkan pemahaman konseptual serta keterampilan analitis mahasiswa.

Kata kunci: Peer Teaching; Media Manipulatif; Pemahaman dan Keterampilan; Matematika Lanjut

Abstract

This study aimed to analyze peer teaching-based learning assisted by manipulative media in improving students' understanding and skills in advanced mathematics courses. The study employed a descriptive quantitative approach along with a qualitative approach, involving 32 students from the PGMI program, semester 3, at the University of Muhammadiyah Bima. The research instrument were the tests to measure understanding and media development skill. Then the second instrument were the questionnaires to measure students' perceptions of the learning process, media development, and understanding. Data were analyzed using descriptive qualitative for the

description of test answers, and descriptive statistics for the questionnaire results. The results showed that students' perception of the peer teaching process assisted by manipulative media was rated as very good (75%), the development of learning media was rated as very good (78.13%), and students' understanding of advanced mathematics material was rated as very good (68.75%). Students' learning outcomes met the graduation criteria with 100% of students achieving the required passing grade, supported by excellent peer teaching skills. The findings concluded that the application of peer teaching assisted by manipulative media can enhance students' motivation and positive perceptions of the mathematics learning process, increase creativity in developing relevant and interactive concrete media, and optimize students' conceptual understanding and analytical skills.

Keywords: Peer Teaching; Manipulative Media; Understanding and Skills; Advanced Mathematics

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INTRODUCTION

Learning Mathematics in elementary schools requires the right techniques and method selection, so that teachers and pre-service mathematics teachers need to understand and master various learning methods and approaches, especially choosing the suitable media that is appropriate to the material. Many studies' results of the use of teaching aids, flat shape media, and counting media provided the increased mathematical understanding in elementary school students (Melisari et al., 2020; Telaumbanua, 2020). Moreover, teacher's perspectives showed that frequent use of manipulative media can improve student learning outcomes in understanding the concepts and procedures of mathematical material (Farhana et al., 2022). In higher education environments, advanced mathematics courses require a deeper understanding of concepts and more complex analytical skills. Advanced mathematics courses require students to have a deep understanding of abstract concepts and the ability to apply them.

However, real-world observations indicate that students often face various challenges in the learning process. This was demonstrated by initial observations

conducted with third-semester students of the Elementary School Teacher Education Program at University of Muhammadiyah Bima, using interviews and discussions that revealed difficulties in understanding mathematical material. Issues faced include difficulties in selecting appropriate media for specific mathematical topics. Students are often confused about which manipulatives to use for explaining certain mathematical concepts. They lack sufficient knowledge about the types, characteristics, and advantages and limitations of each medium, leading to suboptimal learning processes.

The correct teaching method can help students understand concepts more deeply (Kurniawati & Nita, 2018; Ulimaz & Yardani, 2022). However, many students lack the experience and knowledge to select methods that align with both the nature of the material and the needs of the learners. As a result, the learning process becomes less interactive, and learning objectives are not fully achieved, leading to inadequate understanding of mathematical concepts. A strong conceptual understanding is the foundation for mastering advanced mathematics (Eisenhart et al., 2020; Safari & Nurhida, 2024). Unfortunately, many students remain focused on procedural aspects (memorizing formulas and algorithms), which prevents a thorough exploration of fundamental concepts. This impedes their ability to analyze, solve problems, and connect the material learned to other contexts.

Students also have limitations in designing teaching materials based on manipulative media. Although manipulative media is considered effective in facilitating mathematics learning, many students are not equipped with the knowledge and skills to design or utilize this media. This skill is crucial for fostering creativity and innovation in the learning process. Aspiring educators, or those who will later work in the field of education, are expected to have reliable teaching skills. In reality, many still lack confidence in teaching in front of the class, are not skilled in delivering material, and struggle with classroom management (Rasna et al., 2023). This results in low-quality learning and impacts less optimal learning outcomes.

One strategy considered effective for addressing this issue is peer teaching, wherein students themselves take an active role in instructing their peers. Through this process, students not only study the material but also develop communication, collaboration, and reasoning skills. A peer teaching model places students at the center of the learning process, as they collectively teach course content to one another. This approach not only nurtures a sense of responsibility and independent learning, but also reinforces conceptual mastery. Research by Hertiavi & Kesaulya (2020) indicates that students who actively participated as both tutors and tutees in small groups experienced improvements in their physics exam scores. These findings affirm that peer-to-peer interaction can serve as an effective vehicle for enhancing comprehension and fostering autonomous problem-solving skills.

Several studies have demonstrated that implementing peer teaching (peer tutoring) positively influences mathematics learning and comprehension across various educational levels. (Erawati et al., 2024) conducted classroom action research involving junior high school students on the Pythagorean Theorem. Over two learning cycles, the percentage of students demonstrating conceptual understanding rose from 68% (moderate category) to 80% (high category). This improvement suggests that peer teaching can foster a deeper and more effective grasp of mathematical concepts. Similarly, (Nuraeni, 2017), who investigated pre-service mathematics teachers, reported comparable outcomes. The average score for the "junior high school mathematics national exam blueprint" comprehension increased from 59.17 in the pre-test to 75.83 in the post-test of the second cycle (an approximate 28.17% increase), reaffirming the efficacy of peer guidance in enhancing diverse subject matter understanding.

Experimental research likewise supports the advantages of this method. Nawaz & Rehman (2017) discovered that peer tutoring strategies significantly influenced high school students' mathematics learning outcomes. By controlling for external variables such as teacher qualifications, socioeconomic status, and initial academic performance, they confirmed that the observed improvement was

indeed attributable to the application of peer teaching. At the university level, Yusup & Sari (2020) compared the peer teaching model with a lecture-based approach in calculus instruction. Their findings revealed that students learning through peer teaching scored significantly higher than those taught via lectures. Hilger & Schmitz (2022) subsequently highlighted the importance of authenticity and relevance of course materials, particularly for engineering students. They employed peer teaching to integrate real-world applications into mathematics lectures, thus enhancing motivation and conceptual understanding through student-to-student interaction among individuals already familiar with the subject matter and context.

In general, these studies demonstrate that peer teaching not only assists students and undergraduates who face challenges in mathematics, but also enhances conceptual understanding and overall learning motivation. Findings by Abdelkarim & Abuiyada (2016) in Oman further confirm the effectiveness of peer tutor-based instruction in improving academic achievement in mathematics. The consistency of these results across various educational contexts—from junior high schools and high schools to universities—highlights the reliability of peer teaching as a strategy to improve both the processes and outcomes of mathematics learning. The combination of active interaction, peer feedback, and approaches tailored to learners' needs underscores the growing significance of peer teaching in mathematics education.

In addition to peer teaching, employing manipulatives is crucial for helping students visualize abstract concepts. Such resources may include physical devices, geometric models, or even digital simulations that offer visual representations of mathematical principles. Winanda et al. (2024) reported that using manipulatives enhances student participation in the learning process. Similarly, Istofany et al. (2024) found that when students utilize interactive simulations in mathematics instruction, their critical thinking skills improve.

In the context of mathematics instruction, several studies have examined the use of manipulatives. Among these, Cahdriyana & Nurnugroho (2023) underscore the necessity of developing instructional media for plane-sided solids and computational thinking skills due to its greater efficiency. From the perspective of manipulative media, both Farhana et al. (2022) and Larbi & Mavis (2016) found that employing manipulatives significantly facilitates students' understanding of mathematical concepts and procedures, as well as improves learning outcomes. Likewise, research by Martiasari & Kelana (2022) and Ardina et al. (2019) demonstrates enhanced comprehension of polyhedron nets and fraction operations when learning activities are supplemented by manipulatives; the integration of Problem-Based Learning (PBL) and Realistic Mathematics Education (RME) further strengthens these effects.

Additionally, studies by Latifa et al. (2022), Ulyani & Qohar (2021) and Sulistyawati et al. (2021) affirm the validity and practicality of manipulative-based puzzle games, trigonometry media, and STEM "Magic Hours," all of which have proven effective for boosting student motivation and learning outcomes. This approach is also suitable for various educational levels, as noted by Illahi et al. (2021) and Isnaniah & Imamuddin (2020), who reported that paper-based manipulatives help students grasp fractions, while rectangular paper media reinforces their understanding of integer operations. Overall, innovations in mathematics education that integrate manipulatives have demonstrated the ability to address students' conceptual challenges, foster deeper comprehension, and promote better academic achievement in a variety of educational contexts.

These studies confirm the effectiveness of manipulative media in facilitating understanding and enhancing mathematics learning outcomes. However, the application of manipulatives is largely focused on elementary to secondary levels or has not yet been combined with certain instructional methods, such as peer teaching. Moreover, advanced mathematics courses at the university level—characterized by more complex subject matter and the need for abstract reasoning—

have rarely been examined in relation to manipulative media use. Consequently, a research gap exists: how can peer teaching supported by manipulative media be effectively implemented to improve college students' understanding and skills in advanced mathematics courses?

The integration of peer teaching and manipulatives is believed to have a synergistic effect in enhancing students' understanding and skills in advanced mathematics courses. Hwang & Hu (2013) reported that students learning mathematical concepts through peer teaching supported by interactive geometry aids exhibited improved exam scores. They also observed an increase in post-instruction learning retention, highlighting the long-term potential of this approach.

Peer teaching is one approach that can be used to address these issues. This approach allows students to share knowledge and experiences, making the learning process more active and collaborative (Asiah, 2016; Hidayah et al., 2021). In this context, the use of manipulative media also becomes an essential element in helping students visualize abstract mathematical concepts (Evans et al., 2011; Syarifuddin et al., 2024). Manipulative media allows students to directly explore, observe, and experiment with a concept, enabling better internalization of the material.

Based on these problems, this study aims to test the effectiveness of peer teaching-based learning with the assistance of manipulative media in improving students' understanding and skills in Advanced Mathematics courses. It is hoped that through the implementation of this strategy, students will develop a better conceptual understanding, the ability to select appropriate media, skills in designing teaching materials, as well as increased confidence and competence in teaching.

METHOD

The study employed an action research design emphasizing the implementation of the peer teaching method and the use of manipulative media. A descriptive quantitative approach, combined with a qualitative perspective, was used to assess the lesson-planning skills of prospective elementary school teachers. This research employed a descriptive quantitative approach supplemented by a

qualitative approach to assess elementary school teachers' skills in designing learning based on interactive media. Through the quantitative approach, data regarding the students' level of understanding were collected in numerical form using questionnaires. The qualitative approach, on the other hand, focused on exploring the factors that influence students' skills in developing interactive learning, by investigating the methods or strategies they use when explaining certain material. Through quantitative analysis, statistical insights into the students' skill levels were obtained, while the qualitative approach provided a more comprehensive understanding related to the context, motivation, and students' perceptions of using interactive media in the learning process.

Research Subjects

This study was conducted in the PGMI Program at Universitas Muhammadiyah Bima, involving 32 third-semester students enrolled in an Advanced Mathematics course during the 2024/2025 academic year. They were selected as participants because they are still at a relatively early stage of their studies and have not yet taught or engaged in teaching practicums. Additionally, while most third-semester students possessed adequate fundamental mathematical knowledge, they generally had lack substantial experiences in applying teaching methods or designing instruction. As a result, the implementation of a peer-teaching strategy supported by manipulatives can be examined more comprehensively to enhance their mathematical understanding and skills.

Instruments and Data Collection

Data collection in this study involved two instruments. The first instrument was a test in the form of essay questions designed to measure students' understanding and skills in providing answers and elaborating on how they would explain the material to students as a teacher. This data was supplemented by observation results during the class sessions, where peer teaching was implemented by dividing the students into groups of three to four members. Each group was tasked with developing teaching modules, instructional materials, and manipulative

media, as well as conducting teaching practice in front of the class. The second instrument involved a questionnaire consisting of 10 statements: 3 statements about students' perceptions of the learning process, 4 about the development of teaching media, and 3 regarding their understanding of the material. The questionnaire was distributed to the students at the end of the session, after all groups had presented their teaching practices in front of the class.

Data Analysis

The quantitative data collected from the questionnaire responses were analyzed using descriptive statistical analysis. The data were categorized and separated based on the components of the learning process, media development, and understanding of the material. The data were analyzed by calculating the total score for each item and determining the average score. Additionally, the total and average scores of each student's answers were analyzed. The level of students' skills in developing learning was determined based on the percentage from the five categories established, as referenced in Table 1.

Table 1. Categorization and Interval Analysis of Questionnaire Results

No.	Skill Level	Interval
1	Excellent	$4 \leq x \leq 5$
2	Good	$3 \leq x < 4$
3	Adequate	$2 \leq x < 3$
4	Poor	$1 \leq x < 2$
5	Very Poor	$0 \leq x < 1$

Additionally, quantitative data were obtained from students' learning outcomes, derived from the scores they achieved on the test. The scores are presented in terms of individual and class-wide learning mastery, with the minimum passing grade set at a score of 61, equivalent to a letter grade of C, and at least 80% of students are expected to achieve this score.

Furthermore, qualitative data were collected from students' answer sheets by conducting a descriptive analysis of their responses, focusing on media selection, explanation methods, and the accuracy of the answers provided. This analysis was

performed on students' responses that were correct, particularly with regard to the selection of different media and the varying explanations given.

RESULT AND DISCUSSION

Study findings are presented based on students' understanding of advanced mathematical content and their skills in conducting the learning process via a peer teaching approach assisted by manipulative media

Results of Students' Perception Measurement Based on Questionnaire Responses

A total of 32 students participated in completing a questionnaire regarding their perceptions after engaging in instruction through peer teaching supported by manipulative media. These perceptions were divided into three categories: students' perceptions of the Peer Teaching process supported by manipulative media, their perceptions of developing manipulative learning media, and their perceptions of understanding Advanced Mathematics content. The measurement results are presented in Table 2.

Table 2. Student Perceptions of Peer Teaching with Manipulative Media

Kategori	Learning Process		Development of learning media		Material understanding	
	∑ Resp.	%	∑ Resp.	%	∑ Resp.	%
Excellent	24	75	25	78,13	22	68,75
Good	7	21,88	7	21,88	9	28,13
Adequate	1	3,13	0	0	1	3,13
Poor	0	0	0	0	0	0
Very Poor	0	0	0	0	0	0

1. Results of the questionnaire on students' perceptions of the Peer Teaching process assisted by Manipulative Media.

Based on Table 2, it can be observed that students' perceptions of the peer teaching process assisted by manipulative media fall within a relatively high range. Of the 32 students involved, 24 students (75.00%) stated that they had a very

positive perception of the learning process, 7 students (21.88%) rated it as good, and only 1 student (3.13%) showed a less favorable perception. The majority of students with very positive perceptions indicate that the implemented peer teaching method effectively facilitated active interaction among participants, increased learning motivation, and helped them better understand the material presented through manipulative media.

The high level of student satisfaction also suggests that this teaching approach is relevant to the learning needs in the advanced mathematics course. Students tend to feel supported by the opportunity to directly practice the material, engage in discussions, and use concrete media to bridge abstract concepts. The fact that only 1 student gave a "sufficient" perception shows that, overall, the teaching method used was quite effective and met the expectations of the learners.

Table 3. Analysis of the Components of the Peer Teaching Learning Process Assisted by Manipulative Media

Aspect	Learning Process Component		
	Item 1	Item 2	Item 3
Total	143	131	134
Average	4,46	4,09	4,19
Total Component Items		12,75	
Average Item Component		4,25	
Category		Excellent	

Table 3 shows that, overall, the peer teaching learning process assisted by manipulative media received an average score of 4.25. Considering the commonly used rating scale (e.g., a scale from 1 to 5), this score is classified as very high and falls under the excellent category. The average score of 4.25 was derived from various components of the learning process evaluation, including students' ability to design learning activities, clarity in explaining the material using manipulative media, communication skills in front of the class, and the effectiveness of feedback provided during the teaching process. The high ratings indicate that students were able to optimally combine their understanding of advanced mathematics content with pedagogical skills, allowing learners to experience more concrete and interactive learning benefits.

2. *Results of the questionnaire on students' perceptions of the development of learning media.*

Based on Table 2, students' perceptions of the development of learning media through manipulative media are overall classified as excellent. This is evident from the data, which shows that 78.13% (25 students) gave very positive responses, while 21.88% (7 students) gave positive responses. In other words, the majority of students highly appreciated the process and results of creating manipulative media. The high ratings of "very good" emphasize that the media developed by the students are considered relevant to learning needs, easy to adapt, and able to facilitate a more concrete and interactive understanding of mathematical concepts.

Table 4. Analysis of the Components of Manipulative Learning Media Development.

Aspect	Learning Process Component			
	Item 4	Item 5	Item 6	Item 7
Total	143	136	136	133
Average	4,5	4,25	4,25	4,156
Total Component Items		17,13		
Average Item Component		4,28		
Category		Excellent		

Table 4 shows that the development of manipulative learning media by the students received an average score of 4.28, which falls into the "excellent" category based on a 1 to 5 rating scale. This score is derived from various assessment indicators or components, such as design creativity, the relevance of the media to the learning material, clarity of usage instructions, and the effectiveness of the media in aiding conceptual understanding. The high average score reflects the students' ability to design, modify, and present manipulative media that align with the learning objectives and context. Moreover, this result indicates that the students have grasped the fundamental principles of media development, including the

importance of incorporating elements of exploration and interaction for the users (learners). Furthermore, the "excellent" category score emphasizes that the students have demonstrated advanced skills in media development, from selecting appropriate materials, designing the appearance or form of the media, to preparing its use scenario in the classroom.

3. *Results of the questionnaire on students' perceptions of material understanding.*

Based on Table 2, the majority of students (68.75% or 22 students) demonstrated a very high level of understanding of advanced mathematics material. This significant percentage reflects that they not only understand the required formulas and procedures but also can explain the relationships between concepts and apply them to various situations. Meanwhile, 28.13% (9 students) fall into the "good" category, indicating that they have mastered most of the material, although they may still need additional practice to deepen their understanding of certain parts. Only 3.13% (1 student) fall into the "sufficient" category, leading to the conclusion that, overall, students' understanding of advanced mathematics is classified as very good.

Table 5. Analysis of the Components of Advanced Mathematics Material Understanding.

Aspect	Learning Process Component		
	Item 8	Item 9	Item 10
Total	136	125	138
Average	4,3	3,91	4,31
Total Component Items		12,47	
Average Item Component		4,16	
Category		Excellent	

Based on Table 5, the average score for components of advanced mathematics comprehension achieved by students is 4.16, classified as "very good" on a scale of 1 to 5. This score represents a composite of various indicators of understanding, such as the clarity with which students explain key concepts, their ability to interconnect different concepts, and their skills in applying theories to new problems. The high average score reflects that students not only memorize formulas

or algorithms but also grasp them deeply, including the rationale behind the use of formulas and the broader relevance of these concepts.

Furthermore, the achievement of 4.16 as a "very good" category score indicates that the majority of students are capable of delivering advanced mathematics material with clear and logical explanations. They also tend to successfully answer questions, provide case examples, and engage in discussions to critically verify the accuracy of concepts.

Student Learning Outcomes and Skills

The data representing student learning outcomes is reflected in the grades achieved by students, which align with the graduation indicators for the course as stipulated by the regulations of the PGMI study program at University of Muhammadiyah Bima. To pass, students must achieve a minimum score of 61 or a letter grade of C, with at least 80% of students meeting this requirement. According to the student grade data, the minimum score achieved was 69, while the maximum score was 92. All students achieved at least a B−, signifying a 100% pass rate in the advanced mathematics course. This outcome indicates that the instructional strategy employed effectively enabled students to meet the expected competencies. It also illustrates an improvement in the quality of students' comprehension of advanced mathematics, ensuring that the minimum passing standards were successfully satisfied.

Moreover, the attainment of a minimum grade of B- suggests that the majority of students not only mastered fundamental concepts but were also able to demonstrate their understanding through analytical skills and problem-solving abilities. This success is likely influenced by the use of interactive teaching approaches, such as group discussions, the integration of manipulative media, and the application of peer-teaching strategies, which encouraged students to engage in collaborative discussions and verify their understanding with peers. As a result, this achievement highlights not only the high level of student success but also

underscores the importance of innovative teaching practices in maintaining the quality of learning outcomes over the long term.

Description of Students' Skills in Selecting Media

The following describes students' skills in selecting manipulative media and explaining how to deliver solutions based on test items provided. To illustrate this, answers from student subjects were selected, as depicted in the illustrations presented in the following image.

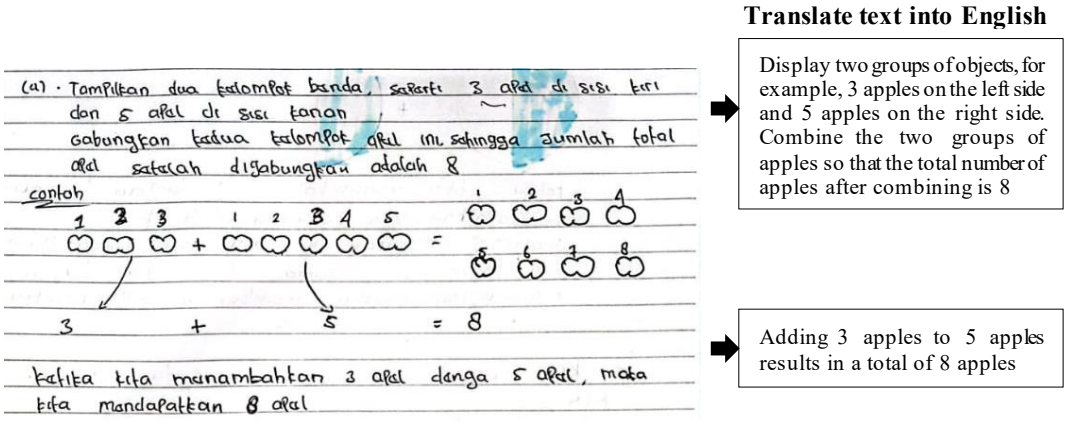

Figure 1. Subject Answer on Media Selection for Addition Illustration

Figure 1 shows that Subject 1 selected apples as a medium to illustrate the operation of addition for 3 + 5. This choice of apples as a medium is highly relevant, as they are not only easily found in everyday life but also serve as a concrete representation that facilitates understanding. Subject 1 explained that to perform the addition of 3 + 5 using apples, the first step is to take 3 apples and place them in one group. Next, Subject 1 takes 5 additional apples and places them in a separate group. Finally, Subject 1 demonstrated that by combining the two groups of apples, the total number of apples becomes 8.

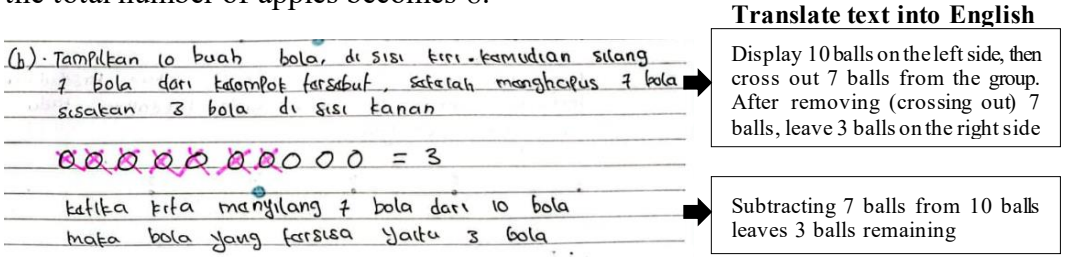

Figure 2. Subject's Answer on Media Selection for Subtraction Illustration

Figure 2 shows that Subject 1 selected balls as a teaching medium to explain the subtraction operation of 10 – 7. In the explanation, Subject 1 began by drawing ten balls to represent the number 10. Subsequently, Subject 1 visually crossed out seven of the balls in the drawing to represent the number 7 being subtracted from 10. This process of crossing out the balls was intended to provide a concrete depiction of subtraction as the act of removing or reducing objects. Through these steps, Subject 1 demonstrated that after crossing out seven balls, only three balls remained, representing the result of 10 – 7. This explanation not only presented the numerical result of 3 but also provided a deeper visual understanding of the subtraction concept, emphasizing the process of reducing the quantity of objects to determine the remainder.

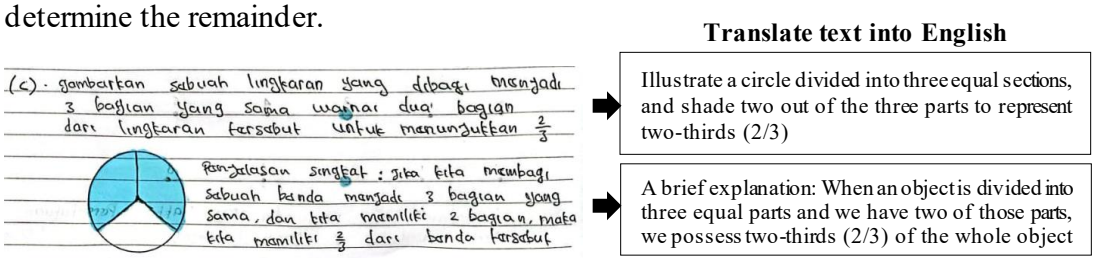

Figure 3. Subject's Answer on Fraction Illustration

The subject chose to use a circle diagram to illustrate the fraction 2/3. The first step was to draw a complete circle and then divide it evenly into three equal parts. This division aimed to demonstrate that the denominator (3) in the fraction 2/3 represents the total number of equal parts. In this case, the three parts of the circle represent one whole unit.

After dividing the circle into three parts, two out of the three parts were marked distinctively, for instance, by coloring them blue. By using a different color, the focus of the learners was directed toward the two colored parts as a representation of the numerator (2) in the fraction 2/3. Through this visualization, learners could clearly see that the two colored sections of the circle represent the selection of two parts from the total of three parts. Adding color to specific sections

also helps learners relate this concept to real-life experiences, where an object can be divided into equal parts, and only a few of those parts are selected or utilized.

Observation Data on Teaching Practice Implementation

Observations during the teaching practice sessions in lectures revealed that student groups successfully designed and applied various types of manipulative media in their teaching practice. They not only utilized common media but also made efforts to modify and innovate based on the mathematical concepts being taught. This process involved stages such as conceptualizing the design, selecting appropriate materials, adjusting shapes and sizes, and testing the use of the manipulative media in simulated teaching scenarios. These efforts demonstrated that the students actively developed their creativity and pedagogical skills. As shown in Figure 4, a variety of manipulative media created by the students were observed, ranging from simple aids like colored paper cutouts to geometric models.

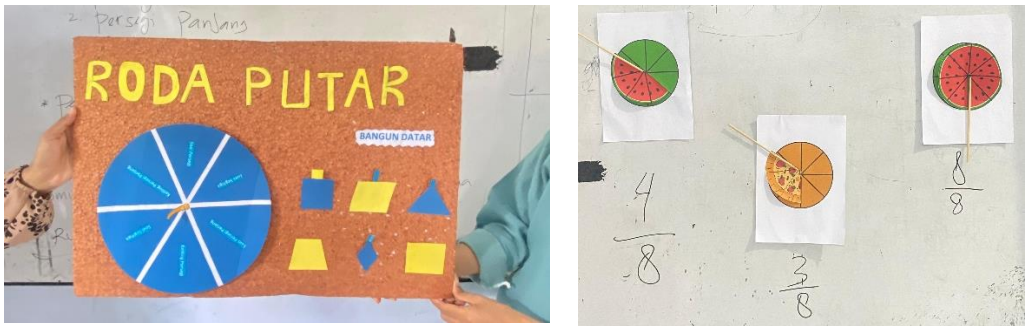

Figure 4. Manipulative Media for Teaching Plane Geometry and Fractions

The results of this study indicate that implementing a peer teaching approach assisted by manipulative media in advanced mathematics lectures successfully enhanced students' understanding and honed their pedagogical skills. This is evident from the students' positive perceptions of the learning process, the development of manipulative media, and their comprehension of the material, all of which were categorized as "very good." The majority of students (75%) expressed a "very good" perception of the learning process, while 78.13% gave a "very good" assessment of the development of manipulative media. This high level of enthusiasm was largely due to the opportunity for students to actively engage in

designing and applying various teaching aids relevant to advanced mathematics material.

Additionally, the students' level of understanding was categorized as "very good," with 68.75% demonstrating deep comprehension and 28.13% categorized as having a "good" understanding. These achievements suggest that the collaborative approach and the use of concrete media effectively bridged abstract concepts, making them easier to grasp (Dewi et al., 2024; Hermawan & Hadi, 2024). In line with this, constructivist theory, which emphasizes the importance of active engagement and exploration by learners, was shown to support improvements in students' conceptual understanding (Aqilla et al., 2024). Meanwhile, the average scores for material comprehension (4.16) and learning process quality (4.25) reflect the consistency of the teaching quality implemented in this course.

On the other hand, learning outcomes, as reflected in students' grades, also demonstrated the success of the teaching strategies employed. All students passed the advanced mathematics course with a minimum grade of B-. This achievement indicates that students not only mastered basic mathematical concepts but were also able to express their understanding through various skills, including analysis and problem-solving. The inclusion of manipulative media in the peer teaching process encouraged students to collaborate, discuss, and validate their understanding (Marlina & Hananto, 2023). Thus, this study underscores that the implementation of peer teaching assisted by manipulative media not only improved students' understanding and skills comprehensively but also provided a solid foundation for developing effective and innovative teaching models for advanced mathematics courses.

The findings of this study align with previous research affirming the effectiveness of peer teaching in improving mathematical understanding and motivation (Nawaz & Rehman, 2017; Nuraeni, 2017; Yusup & Sari, 2020). As concluded by (Erawati et al., 2024), implementing peer teaching can strengthen students' conceptual understanding of mathematics, especially when integrated

with manipulatives. The emphasis on using tangible materials is further supported by Farhana et al. (2022), Larbi & Mavis (2016), and Martiasari & Kelana (2022), who observed that manipulative media facilitate visualization and reduce conceptual errors. In addition, the studies by Latifa et al. (2022), Ulyani & Qohar (2021), and Sulistyawati et al. (2021) highlight the importance of developing concrete media aligned with contemporary needs, including STEM approaches, puzzle-based games, and various other manipulatives to enhance students' creativity and analytical skills. Consequently, the latest findings—indicating a "highly positive" category in student perceptions, media development, and conceptual understanding—expand existing evidence that combining peer teaching with manipulative media can address an array of challenges in mathematics education, even at the college level, where more advanced conceptual and analytical abilities are required.

Despite these positive outcomes, several limitations should be noted. First, the relatively small sample of 32 students from a single study program at Universitas Muhammadiyah Bima may restrict the diversity of participant characteristics. Such constraints mean that differences in academic background, motivation, or initial understanding may not be fully represented. Furthermore, relying on only one institution may limit the replicability of the results, as cultural and curricular contexts may vary in other higher education settings. Contextually, the research results can only be generalized to subjects under the same exact conditions (Kusumastuti et al., 2024).

Another potential source of bias involves participant selection and methodology. Students who are more motivated or willing to engage actively in peer teaching may possess higher levels of enthusiasm and teaching aptitude than others, potentially skewing the findings toward positive outcomes without reflecting the broader student population. In accordance to research result, it shows that students who have equipped themselves with teaching knowledge and theory can practice microteaching very well (Astuti, 2023). Additionally, variations in

instructor and peer tutor skill levels, as well as the availability of time, may also influence the quality of peer teaching and thus introduce inconsistencies in its effectiveness.

In terms of implementing peer teaching assisted by manipulative media, technical and pedagogical obstacles may arise, such as limited student knowledge of designing and using suitable manipulative tools for specific topics. If learning materials are not developed appropriately, instruction could be less optimal despite the peer teaching format. Moreover, differences in communication skills, confidence, and group management among peer tutors can significantly influence the success of the learning process.

These limitations affect the extent to which the findings can be generalized. The results may not immediately translate to different educational contexts, course characteristics, or institutional cultures. It is therefore recommended that future studies replicate these methods with larger samples, across multiple institutions, and under stricter controls to strengthen external validity. By doing so, the efficacy of peer teaching supported by manipulative media in enhancing students' understanding and skills in Advanced Mathematics courses can be confirmed more convincingly and adapted to a wider range of educational settings.

CONCLUSION

The findings of this study and the discussion indicate that students had a "very good" perception of the peer teaching process assisted by manipulative media. Additionally, students' perceptions of the development of manipulative teaching media were also categorized as "very good." The level of understanding of advanced mathematics material was similarly categorized as "very good." Therefore, this study concludes that the implementation of peer teaching assisted by manipulative media was able to (1) foster motivation and positive student perceptions of the mathematics learning process, (2) enhance creativity in developing relevant and interactive concrete media, and (3) optimize students' conceptual understanding and analytical skills. The majority of students met the

graduation criteria with grades that met the required standards, confirming the success of this teaching method in fulfilling the course competencies. This success also highlights the importance of collaborative and experiential approaches in fostering a deep understanding of mathematics. Thus, the findings of this study are expected to serve as a reference for the development of other innovative teaching models that can continue to maintain and improve the quality of mathematics learning outcomes in higher education.

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