ISSN 2303-0992
ISSN online 2621-3176
**Matematika dan Pembelajaran**
Volume 13, No. 1, June 2025, h. 19-42
**Submitted**: April 17, 2025; **Revised**: May 19, 2025; **Accepted**: June 9, 2025

# HYPOTHETICAL LEARNING TRAJECTORY BASED ON INQUIRY LEARNING TO FIND THE VOLUME OF SPACE

**Friska Alifia Rizki[1], *)Mohammad Faizal Amir[2]**
*[1,2]Primary Teacher Education, Universitas Muhammadiyah Sidoarjo, Indonesia*
**)Corresponding author*
faizal.amir@umsida.ac.id

## Abstrak

Geometri merupakan cabang ilmu dalam mata pelajaran matematika yang berperan penting dalam pengembangan kemampuan berpikir siswa. Studi internasional dan nasional menunjukkan siswa memiliki pemahaman rendah terhadap konsep geometri, khususnya volume bangun ruang. Hal ini disebabkan oleh pembelajaran yang tidak memfasilitasi untuk membangun pemahaman siswa. Studi ini bertujuan untuk mengembangkan dan mengimplementasikan *Hypothetical Learning Trajectory* berbasis *Inquiry Learning* (HLT-IL) untuk memfasilitasi pemahaman siswa terhadap volume kubus dan balok. Studi ini menggunakan metode design research yang terdiri dari tiga tahap utama: mempersiapkan eksperimen, merancang eksperimen, dan analisis retrospektif. Instrumen penelitian terdiri dari lembar kerja, lembar observasi, dan tes. Subjek studi adalah siswa kelas lima, yang dikategorikan berdasarkan tingkat kemampuan: rendah, sedang, dan tinggi. Studi ini menghasilkan lima aktivitas HLT-IL untuk membangun pemahaman konsep volume bangun ruang: orientasi, konseptualisasi, investigasi, kesimpulan, dan diskusi. Dalam hal ini, terdapat sembilan sub aktivitas HLT-IL: pengenalan dan penemuan, pengajuan pertanyaan, pembuatan hipotesis, eksplorasi, eksperimen, interpretasi data, merangkum dan membandingkan, komunikasi, dan refleksi. Hasil studi menunjukkan bahwa dari 26 siswa (92,86%) dalam kategori memuaskan dan 2 siswa (7,14%) dalam kategori kurang memuaskan. Hasil ini membuktikan bahwa lintasan pembelajaran berbasis inkuiri dapat menjadi alternatif pembelajaran di sekolah dasar yang bertujuan untuk menemukan suatu konsep, khususnya volume bangun ruang.

**Kata kunci:** Inkuiri; Lintasan Pembelajaran; Volume Bangun Ruang

## Abstract

Geometry is a branch of mathematics crucial in developing students' thinking skills. International and national studies show that students have a low understanding of geometry concepts, especially the volume of space. It is caused by learning that does not facilitate building students' understanding. This study aims to develop and implement a Hypothetical Learning Trajectory based on Inquiry Learning (HLT-IL) to facilitate elementary students' understanding of the volume of cubes and blocks. A design research methodology comprised three main phases: preparing for the experiment, conducting the teaching experiment, and retrospective analysis. The study's instruments consisted of worksheets, observation sheets, and tests. The participants were fifth-grade students categorized into three skill levels: low, medium, and high. The resulting HLT-IL comprised five main activities: orientation, conceptualization, investigation, conclusion, and discussion, and nine sub-activities: introduction and discovery, questioning, hypothesis generation, exploration, experimentation, data interpretation, summarizing and comparing, communication, and reflection. The results showed that 26 out of 28 students (92.86%) reached the satisfactory category, while 2 students (7.14%) remained in the unsatisfactory category. These findings indicate that the inquiry-based learning trajectory can be an effective alternative for supporting conceptual discovery, particularly in learning the volume of space in primary education.

**Keywords**: Hypothetical Learning Trajectory; Inquiry Learning; Volume of Space

**Citation:** Rizki, A.F., Amir, M.F. 2025. Hypothetical Learning Trajectory Based on Inquiry Learning to Find Volume of Space. *Matematika dan Pembelajaran*, 13(1), 19-42. DOI: http://dx.doi.org/10.33477/mp.v13i1.9303

## INTRODUCTION

Geometry is one of the core branches of mathematics. It serves as a bridge between theoretical concepts and real-world applications (Ticu, 2023). Through learning geometry, students can understand and analyze various shapes and spatial objects in their environment (Chan & Leung, 2014), ranging from two-dimensional figures such as squares and circles to three-dimensional objects such as cubes and spheres. Thus, geometry plays a vital role in education, offering conceptual knowledge and practical relevance in everyday life.

One of the key topics in geometry is the volume of space, which involves the dimensions of height, width, and length (Sahria & Yulfihani, 2023). Understanding volume, such as in cubes and rectangular prisms, is essential in primary mathematics as it fosters spatial visualization and comprehension of three-dimensional structures (Seah & Horne, 2020). Primary students often learn about volume through visual models representing the number of unit cubes that fill a given space (Putri, 2019). Thus, learning volume promotes engagement and builds a strong foundation for mathematical reasoning and exploration.

According to the 2022 PISA results, Indonesian students ranked 65th out of 81 countries in mathematics, with geometry scores averaging 372, well below the international average of 494 (OECD, 2023). This highlights the persistent low understanding of geometry among Indonesian students. In classrooms, many teachers still apply conventional methods that limit students' opportunities to construct or discover geometric concepts, especially volume (Sisman & Meral, 2016). As a result, students often lack conceptual understanding (Hisyam et al., 2021) and face learning difficulties (Rozgonjuk et al., 2020).

The Hypothetical Learning Trajectory (HLT) is a conceptual model for mathematics instruction design. It outlines a hypothetical path that students might take toward understanding a particular concept and consists of three core components: a learning goal, a set of learning tasks, and a hypothesized learning process (Simon, 2020). For instance, Mutaqin et al. (2019) developed an HLT to support students' understanding of value comparison through the real-life context of Pandanwangi rice farmers in Cianjur. The learning goal focused on developing students' proportional reasoning. The learning tasks involved analyzing agricultural data, while the learning process guided students from contextual comprehension toward formal mathematical abstraction.

Although HLT has been shown to improve student understanding (Clements, 2020; Labibah & Amir, 2022). Studies suggest its effectiveness can be enhanced by integrating other instructional approaches, such as virtual learning (Mohseni et al., 2024) and realistic mathematics education (Neno et al., 2023). One such approach is Inquiry Learning (IL), which promotes active engagement through hypothesis formulation, experimentation, and observation. (Haas, 1995). Like scientists, students investigate problems, gather data, and draw conclusions. For instance, Triwahyuningtyas & Suastika (2019) guided students in exploring a sphere's surface area and volume using peeled oranges and real object comparisons, highlighting IL's discovery-based nature.

Despite these advancements, most studies have explored HLT, IL, and volume of space as separate topics. Studies on HLT have focused on specific contexts. Such as Tunimah et al. (2024) realistic mathematics education with digital animation, Nursyahidah & Albab (2021) ethnomathematics, Suciana et al. (2020) real-life applications, Jais et al. (2023) or van Hiele's theory, but it is rarely integrated with IL. On the other hand, IL has been Rahman & Halim (2024) through augmented reality or Feriana (2016) engineering design projects without the HLT framework. Other studies by Rahmatin et al. (2019), Sanwidi (2020), and Mashuri (2020) have focused solely on media use without connecting to HLT or IL. No previous study has simultaneously integrated HLT and IL (HLT-IL) in the context of learning about volume.

The integration of HLT and IL offers a key advantage for students: it supports the gradual and meaningful construction of conceptual understanding. HLT provides a structured learning sequence, while IL promotes active engagement through questioning, exploration, experimentation, and reflection. Together, they help students develop both conceptual insight and critical thinking skills (Nugraha et al., 2023). This integration is also a practical guide for teachers in designing learning to facilitate meaningful concept formation that is responsive to differences in student skills. Thus, this study aims to bridge this gap by designing and implementing an HLT-IL model to support students' conceptual understanding of volume through structured, active exploration.

The HLT-IL learning design needs further exploration as a meaningful approach to address students' difficulties in understanding the concept of volume. Many students struggle to connect concrete representations with abstract ideas, particularly in relating length, width, height, and volume (Sahria & Yulfihani, 2023; Seah & Horne, 2020). Through structured activities that encourage questioning, exploration, and independent reasoning, HLT-IL fosters critical thinking and gradual concept formation. This study is therefore essential to introduce a learning model that is not only teacher-guided but also responsive to students' needs. The HLT-IL framework is expected to help primary students meaningfully explore the relationships among length, width, and height, enhancing their conceptual understanding of volume. Thus, this study aims to develop and implement the HLT-IL model to support that goal.

## METHOD

### *Research Design*

This study employed design research following the framework proposed by Akker et al. (2011), which consists of three main stages: (1) preparing for the experiment, (2) designing the experiment, and (3) conducting a retrospective analysis. Design research was used to develop a Hypothetical Learning Trajectory based on Inquiry Learning (HLT-IL) that facilitates primary students' conceptual understanding of the volume of space. The cycle of the design research implemented in this study is illustrated in Figure 1.

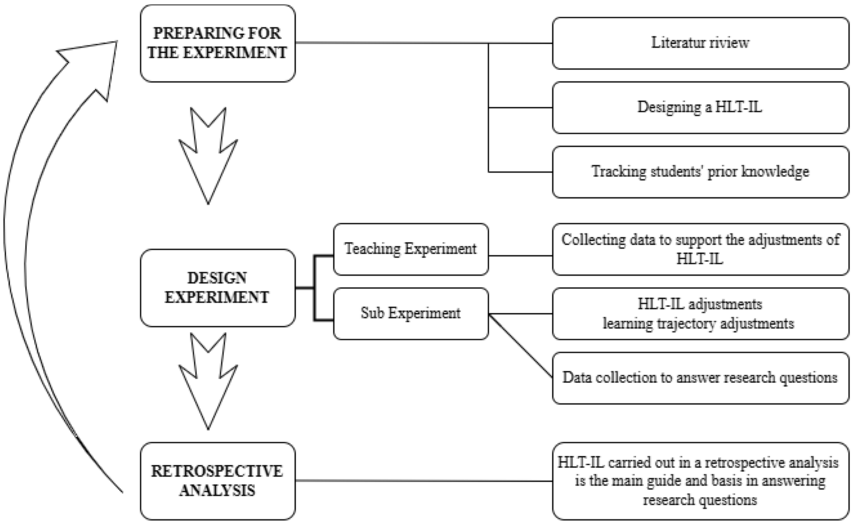



**Figure 1. Cycle of Design Research**

Figure 1 illustrates the design research cycle used to develop and test the HLT-IL based on inquiry learning for teaching volume. The first stage, preparing for the experiment, involved a literature review, designing the HLT-IL, and identifying students' prior knowledge. The second stage, designing the experiment, included teaching experiments and sub-experiments to implement and evaluate the HLT-IL. Data collected during this phase informed revisions to align learning activities with inquiry-based principles. The final stage, retrospective analysis, focused on analyzing data to refine the HLT design and address the research questions. This process was iterative, enabling continuous improvement of the learning trajectory.

***Participants***

This study involved 28 students at Punggul State Primary School, Gedangan, Sidoarjo. Participants were selected using a purposive sampling technique (Creswell, 2018). Students' initial mathematical skill was determined based on their scores from three daily mathematics tests. These scores were categorized into three skill levels: low, medium, and high (Ayuningtyas et al., 2024). The results of this categorization are presented in Table 1.

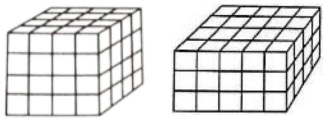
**Table 1. Categorization Results of Students' Mathematics Levels**

| Categories | Score intervals | Total | Average Score |
|---|---|---|---|
| Low | $0 \leq x < 75$ | 6 students | 72.22 |
| Medium | $76 \leq x < 80$ | 9 students | 85.18 |
| High | $81 \leq x < 100$ | 13 students | 94.92 |

***Instruments and Data Collection***

This study's instruments and data collection techniques included tests, worksheets, and observation sheets. The data were gathered from observations made during the learning process using an observation sheet adapted by Amir & Wardana (2017), a worksheet developed by Amir (2018), and a test instrument based on Rofieq (2008). To ensure the validity in this study, the design of HLT-IL and the whole instrument, including worksheets, student activity observation sheets, and tests, were validated through a two-stage process. First, expert validation was conducted by a mathematics education lecturer to assess the suitability of the content, the integration between activities, and the feasibility of HLT-IL steps. Second, a class teacher conducted a readability test to ensure that students can understand the instrument's language, context, and instruction according to their level. Indicators and sample items from the worksheet are presented in Table 2.

**Table 2. Indicators and Sample Items**

| Indicators | Sample Items |
|---|---|
| 1. Able to arrange and illustrate the shapes of cubes and blocks using unit dice. |  1. Arrange the dice into cube and block shapes as instructed! |
| 2. Able to describe the shape of the spaces that have been arranged. | 2. Draw the cubes and blocks you have made! |
| 3. Able to calculate the number of dice used to form the shapes. | 3. Count the number of dice used to form the cubes and blocks! |
| 4. Able to determine the volume of the shape based on the number of dice used. | 4. Determine the volume of the cube and block based on the number of dice you have used! |
| 5. Able to measure the length, width, and height of the shapes made. | 5.1 Measure the cubes and blocks' length, width, and height from other groups!<br>5.2 Are the cube's length, width, and height the same? Explain!<br>5.3 Are the block's length, width, and height the same? Explain! |
| 6. Able to analyze the relationship between length, width, and height with the volume of the shape. | 6. Explain the relationship between length, width, height, and the volume of a shape! |
| 7. Able to conclude the formula for the volume of cubes and blocks based on the investigation results. | 7. Based on your investigation, write down the formula for the volume of cubes and blocks! |

The study procedure followed the stages of design research, beginning with the preparation for the experiment, which comprised three main cycles. First, a needs analysis was conducted through a literature review on Hypothetical Learning Trajectories (HLT) (Wijaya et al., 2021) and Inquiry Learning (IL) (Artigue et al., 2020; Pedaste et al., 2015). Second, students' initial mathematical abilities were assessed using results from three daily tests. Third, the HLT-IL design was developed by integrating HLT components—objectives, activities, and hypotheses (Simon, 2020)—with the five IL phases: orientation, conceptualization, investigation, conclusion, and discussion (Artigue et al., 2020; Pedaste et al., 2015). The design included conjectures of student responses across low, medium, and high skill levels. Table 3 presents the objectives and activities of HLT-IL, while Table 4 outlines the response conjectures by student category.

**Table 3. Design of HLT-IL**

| IL Activities | IL Sub Activities | IL Sub Activities in Cube and Block Volume |
|---|---|---|
| Orientation | Introduction and discovery | Creating the cube and block shapes of the right size using dice. |
| Conceptualization | Questioning | Formulate a question on how to count the volume of space. |
| | Hypothesis generation | Counting the dice that make up the cube and block. |
| Investigation | Exploration | Counting the side length, width, and height of the cube and the block. |
| | Experimentation | Counting the number of dice from the side length, width, and height parts of the shape cube and block from other groups. |
| | Data interpretation | Analyzing data collection results and connecting them with the volume concept. |
| Conclusion | Summarize and Compare | Summarizing with the volume formula based on the observation result and analysis that have been done. |
| Discussion | Communication | Discussing the various ways of counting volume and difficulty encountered critically. |
| | Reflection | Evaluating students' understanding of the volume concept. |

**Table 4. Conjectures of HLT-IL per IL Sub Activities**

| IL Sub Activities | Conjectures of HLT-IL in Cube and Block Volume |
|---|---|
| Introduction and discovery | – High-skilled students can figure out the shape and size of cubes and blocks of the right size using unit cubes (dice) without needing help.<br>– Medium-skilled students can only create shape cubes and blocks using unit cubes (dice) after being given instructions and examples.<br>– Low-skilled students need more extended time and more intensive assistance to create shapes and sizes and build a space cube and block. |
| Questioning | – High-skilled students can formulate complex questions about how to count the volume of built space.<br>– Medium-skilled students can only formulate questions about the definition of volume and build space.<br>– Low-skilled students will have difficulty formulating questions related to volume concepts and are more likely to ask factual questions. |
| Hypothesis generation | – High-skilled students can determine the number of unit cubes (dice) in building a space cube and block without needing help.<br>– Medium-skilled students can only determine the number of unit cubes (dice) in building a space cube and block after being given instructions and examples. |

| | |
|---|---|
| | – Low-skilled students will experience difficulty only counting and determining the number of unit cubes (dice) on the surface of the build-a-space cube and block. |
| Exploration | – High-skilled students can determine the size of the side length, width, and height in building a space cube and block correctly without needing help.<br>– Medium-skilled students are only able to measure the visible side.<br>– Low-skilled students will have difficulty distinguishing between side length, width, and height when building a space cube and block. |
| Experimentation | – High-skilled students can determine the amount of volume by measuring the side length, width, and height of a space cube and a block of various sizes.<br>– Medium-skilled students can only measure a space cube and block's length, width, and height.<br>– Low-skilled students will struggle to distinguish between side length, width, and height when building a space cube and block. |
| Data interpretation | – High-skilled students can determine the relationship between the measurement of the side length, width, and height with the volume of the building space.<br>– Medium-skilled students can only determine the built space's side length, width, and height measurements.<br>– Low-skilled students will experience difficulty in analyzing data collection. |
| Summarize and compare | – High-skilled students can conclude and determine the volume of cube and block formulas based on the observation result and data analysis that have been done.<br>– Medium-skilled students are only able to conclude about the volume of a building based on the observation result and data analysis that have been done.<br>– Low-skilled students will experience difficulty in coming to a conclusion based on the observation result and data analysis that have been done. |
| Communication | – Students are expected to be able to actively participate in critical discussions to find a solution regarding the obstacles faced after carrying out the experimental process. |
| Reflection | – Students are expected to be able to realize mistakes in their understanding. |

The second stage was the design of the experiments, which consisted of two cycles. First, the initial trial of the HLT-IL design aimed to evaluate its feasibility within the learning context. At this stage, the HLT-IL design was implemented in a limited setting to identify its strengths and weaknesses. Based on the findings from this trial, revisions were made to improve the design by addressing shortcomings in teaching strategies and instructional steps, with the expectation of enhancing its overall effectiveness. Second, the revised HLT-IL was applied in a broader context of implementing experimental learning. At this stage, data were collected to evaluate how the developed design supported improving students’ conceptual understanding.

The third stage was the retrospective analysis. In this phase, researchers compared the hypothesized HLT-IL with the learning process observed during the experimental implementation. This analysis focused on identifying the underlying causes of the observed outcomes and determining areas for improvement. It is important to note that design research does not aim for immediate success but seeks to understand how and why an intervention works effectively.

***Data Analysis***

Data analysis was conducted using triangulation to enhance the validity of the findings (Miles, 2014), by integrating data from worksheets, observation sheets, and tests. Worksheets were analyzed to evaluate students' thought processes, while observation sheets documented students' engagement across five activity categories: Orientation and Discovery (OR), Conceptualization (CP), Investigation (IN), Conclusion (CL), and Discussion (DI). Tests were administered to assess students' understanding of the concept of volume of space. All three data sources were comprehensively analyzed during the retrospective analysis stage to evaluate the effectiveness of the HLT-IL design and to refine the learning trajectory, ensuring it is more responsive to students' learning needs.

## RESULT AND DISCUSSION

The study's results on the volume of space learning trajectory were used to determine the impact of the Hypothetical Learning Trajectory integrated with Inquiry Learning (HLT-IL) on students' conceptual understanding of deriving volume formulas for cubes and blocks. The implemented learning trajectory comprised five sequential activities: orientation, conceptualization, investigation, conclusion, and discussion.

### *Observation Results*

Classroom observations were conducted before analyzing the HLT-IL implementation's outcomes. These observations aimed to capture the learning dynamics and students' engagement throughout the activities. Each activity within the trajectory was assessed to identify the students' levels of understanding within their respective groups during the implementation of HLT-IL. The results are summarized in Table 5.

**Table 5. Observation Results on IL Sub-Activities**

| Group | Coding of IL Sub Activities | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | OR1 | CP1 | CP2 | IN1 | IN2 | IN3 | CL1 | DI1 | DI2 |
| Group 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Group 2 | 1 | 3 | 2 | 3 | 3 | 3 | 3 | 3 | 2 |
| Group 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Group 4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 3 |
| Total | 7 | 12 | 11 | 12 | 12 | 12 | 12 | 11 | 11 |
| Average (%) | 58.3 | 100 | 91.7 | 100 | 100 | 100 | 91.7 | 91.7 | 91.7 |

Descriptions:
OR1 = Orientation in introduction and discovery
CP1 = Conceptualization in questioning
CP2 = Conceptualization in hypothesis generation
IN1 = Investigation in exploration
IN2 = Investigation in experimentation
IN3 = Investigation in data interpretation
CL1 = Conclusion in summarize and compare
DI1 = Discussion in communication
DI2 = Discussion in reflection

Table 5 shows that student participation was highest (100%) in activities CP1, IN1, IN2, and IN3, indicating strong engagement in question generation and investigation. Activities such as CP2, CL1, DI1, and DI2 also had high involvement (91.7%), reflecting students' ability to formulate hypotheses, draw conclusions, and participate in discussions. However, the lowest participation occurred in OR1 (58.3%), suggesting limited engagement during the initial orientation phase. These findings highlight that investigation and conceptualization were the most engaging phases, while orientation needs improvement to better capture students' interest at the start of learning.

### *Orientation Activity*

In the orientation activity, students were introduced to volume through direct, hands-on experiences using dice as physical manipulatives. Students were encouraged to observe and actively explore the given problem. Observational data showed that some student groups were enthusiastic and highly focused while arranging dice to form three-dimensional shapes, as illustrated in Figure 2.

**Figure 2. Students Construct a Space Using Dice**

High-skilled students quickly understood the instructions and arranged the shapes with accurate proportions, while medium and low-skilled students required more time and corrections. This activity stimulated the emergence of questions and hypotheses related to calculating volume based on the arrangement of the dice. Evidence of worksheet collaboration appeared during discussions among students before completing the worksheet, as shown in Figure 3.

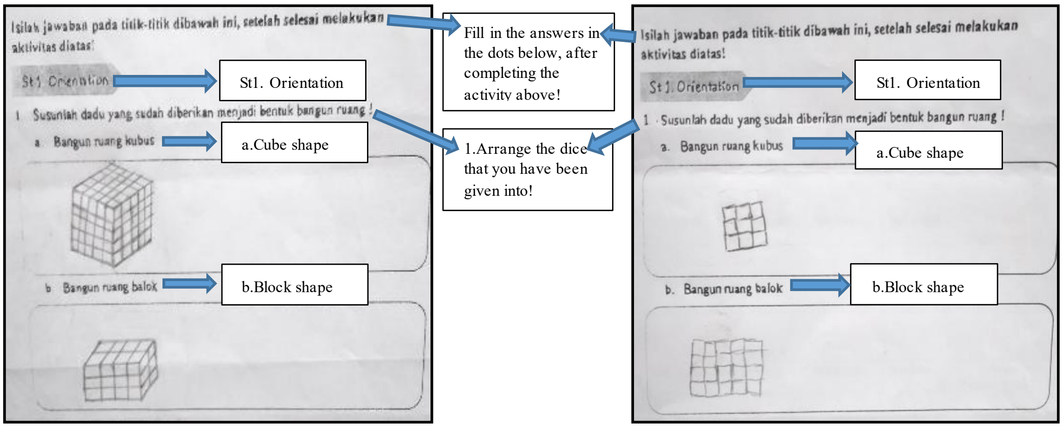



**Figure 3. (a) Correct Answers; (b) Incorrect Answers in Visualising**

Group 4 successfully arranged the dice into cube and block shapes using the requested models. The top arrangement accurately formed a cube in quantity and structure, while the bottom arrangement resembled a space block. In contrast, Group 2 did not successfully construct the expected shapes; their dice arrangement failed to resemble either cubes or blocks.

### *Conceptualization Activity*

In the conceptualization activity, students participated in two cycles. In the first cycle, they posed questions related to volume, such as its definition, causes of volume differences, and methods to calculate the volume of cubes and blocks. Some students explored the distinction between volume and area or sought to understand volume beyond relying on memorized formulas. High-skilled students demonstrated strong reasoning by formulating critical questions and relevant hypotheses based on dice arrangements. Meanwhile, medium- and low-skilled students required teacher or peer support to construct meaningful hypotheses.

In the second cycle, focused on hypothesis generation, students explored volume using unit dice. The teacher prompted curiosity with questions like, “How do we determine the space occupied by the unit dice?” Students responded with hypotheses such as direct counting or identifying arrangement patterns. High-skilled students recognized systematic strategies, medium-skilled students proposed ideas but needed help identifying patterns, and low-skilled students required more concrete examples and direct guidance.

### *Investigation Activity*

In the investigation activity involving three cycles, students carried out structured group tasks to deepen their understanding of volume. First, during the exploration cycle, students collaboratively measured the length, width, and height of cube and block constructions using unit dice and documented their findings (Figure 4). High-skilled students measured independently and accurately, linking data to volume concepts. In comparison, medium- and low-skilled students focused on visible dimensions and needed support to interpret measurements.

**Figure 4. Students' Exploration Through Observing the Number of Dice**

Second, in the experimental cycle, each group exchanged its constructed shapes for measurement. Students measured dimensions and discussed to verify the results. High-skilled students connected these results to the concept of volume, while medium- and low-skilled students still relied on guidance to grasp the relationships involved. Third, in the data interpretation cycle, students analyzed dimensional relationships to determine volume, examined the effects of changing dimensions, and drew initial conclusions as a basis for formulating volume. High-skilled students logically identified patterns and variable relationships, whereas others needed further support to engage in this analytical process.

***Conclusion Activity***

In the conclusion activity, students formulated generalizations regarding volume in spatial figures, particularly cubes and blocks, using unit dice. They recorded their findings in worksheets while engaging in group discussions to ensure mutual understanding. Student interactions during this phase reflected collaborative learning and active involvement in deriving conceptual conclusions, as illustrated in Figure 5.

**Figure 5. Group Discussion to Find the Volume Formula of Cube and Block**

High-skilled students successfully concluded the volume formula based on their observations and analysis and were able to compare their conclusions with those of other groups. In contrast, medium and low-skilled students required teacher guidance to identify data patterns and derive appropriate conclusions. This process served as a synthesis of prior observations and experiences from the investigation activity. Examples of students' responses during the activity are shown in Figure 6.

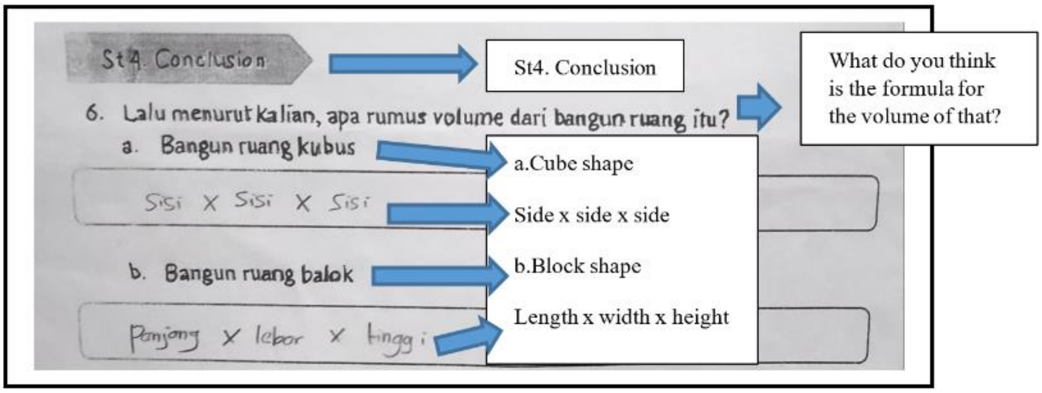



**Figure 6. Students' Answers in the Conclusion Activity**

Students constructed their understanding of the volume formula based on prior exploration and analysis and were subsequently asked to formalize their conclusions. For cube-shaped figures, students formulated the volume as the product of side × side × side. For block-shaped figures, they expressed volume as the product of length × width × height.

***Discussion Activity***

In the discussion activity, two cycles were carried out. The first communication activity involved students in critical discussions about their findings and challenges during the experiment. They shared results, raised questions, and collaboratively developed evidence-based solutions. The second, reflection activity, focused on evaluating their learning processes through peer or teacher-guided discussions of answers, strategies, and conceptual understanding. Students identified and corrected reasoning errors with the help of teacher facilitation and peer feedback. The final HLT structure derived from the retrospective analysis is shown in Table 5.

**Table 5. HLT as a Result of Retrospective Analysis**

| Objective | Students' Thinking in the Actual Classroom |
|---|---|
| Orientation | – High-skilled students could correctly make and draw shapes of cubes and blocks.<br>– Medium and low-skilled students can make shapes of cubes and blocks until they complete the task, but they need a longer time and some corrections to ensure the shapes are appropriate. |
| Conceptualization | – High-skilled students can formulate accurate, critical questions and hypotheses related to the volume concept of building space.<br>– Medium and low-skilled students need guidance to formulate questions and hypotheses. |
| Investigation | – High-skilled students could count and analyze the data correctly.<br>– Medium and low-skilled students are counted on the side only and need guidance in analyzing data. |
| Conclusion | – High-skilled students were able to conclude the volume formula correctly and critically from the observation result and data analysis that had been done.<br>– Medium and low-skilled students need direction for the volume formula. |
| Discussion | – High-skilled students actively discuss and give in-depth reflection.<br>– Medium and low-skilled students give simple opinions in discussion and reflection. |

Based on the retrospective analysis, the HLT-IL approach revealed variations in student understanding at different stages. High-skilled students demonstrated greater independence in constructing concepts, while medium-skilled students required support when formulating questions and drawing conclusions. Low-skilled students needed intensive scaffolding, especially in the orientation and data interpretation activity. Scaffolding was provided through direct guidance when arranging the cube of unit dice, guiding questions during the experiment, and visual assistance to understand the relationship between the dimensions of length, width, and height to volume. These findings highlight the need for differentiated instruction, especially during visualization and manipulative activities. The HLT-IL implementation concludes with a test to assess students' conceptual understanding.

***Test Results***

The implementation of HLT-IL concluded with a test designed to assess students' understanding. Students were evaluated and the test results were categorized into two groups: satisfactory and unsatisfactory. The satisfactory category indicated that students had developed a good understanding of the material, whereas the unsatisfactory category reflected a lack of conceptual comprehension. The distribution of these results is illustrated in Figure 7.

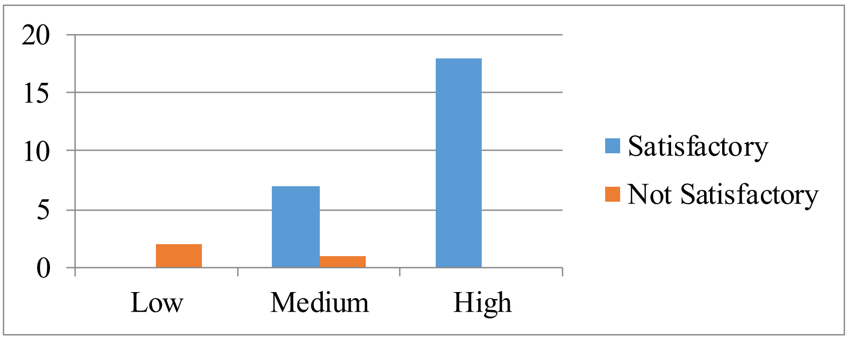



**Figure 7. Students' Test Score Results**

The test results indicated an improvement in students' understanding after implementing HLT-IL. Of the 28 students, 26 (92.86%) achieved scores within the satisfactory category (score ≥ 51), while 2 students (7.14%) remained in the unsatisfactory category (score < 51). All high-skilled students (scores 76–100) demonstrated independent conceptual understanding. Most medium-skilled students (scores 51–75) also reached satisfactory levels, whereas low-skilled students (scores 0–50) continued to require additional instructional support.

The first key finding reveals that HLT-IL comprises five main activities and nine sub-activities, forming a structured and systematic learning trajectory. This aligns with the perspective of Clements (2020) who underscores the significance of explicit, exploration-based learning paths. Similarly, Gravemeijer & Doorman (1999), argue that explorative and discursive activities are essential for constructing mathematical meaning. However, in contrast to Nugraha et al. (2023), who emphasized the importance of the orientation stage, this study identified orientation as a relatively weak component, potentially due to students' unfamiliarity with concrete spatial representations. Furthermore, van Bemmel et al. (2023) highlighted that the teacher's skill significantly influences the effectiveness of HLT in responding to classroom dynamics. This is reflected in the observed necessity for differentiation according to students' skill levels. These findings suggest that the design of HLT-IL should remain flexible and adaptive to accommodate variations in students' readiness and background knowledge.

In the orientation activity, students were introduced to the concept of volume of space using unit dice as representations of cubes and blocks. High-skilled students dominated this stage, as they were able to quickly comprehend the instructions and effectively bridge concrete experiences with abstract mathematical concepts (Putri, 2019). In contrast, medium and low-skilled students required scaffolding to support their understanding of constructing the intended cube and block shapes (Saputra et al., 2024). During the conceptualization activity, high-skilled students demonstrated the skill to formulate complex questions and hypotheses, reflecting their advanced critical thinking skills (Nugraha et al., 2023). Meanwhile, medium and low-skilled students struggled to generate meaningful questions and required guidance due to their limited conceptual understanding.

In the investigation activity, high-skilled students were able to measure and analyze data accurately, connecting it to volume concepts through their strong representational skills (Maisyarah & Prahmana, 2020). In contrast, medium and low-skilled students displayed misconceptions by measuring only the visible sides, thereby indicating the need for scaffolding to support understanding (Haidar et al., 2020). During the conclusion activity, high-skilled students successfully formulated, whereas medium and low-skilled students required teacher support to articulate even basic conclusions. In the discussion activity, high-skilled students confidently conveyed their understanding of volume concepts, which is attributed to their higher self-efficacy (Rozgonjuk et al., 2020). Meanwhile, their peers benefited from collaborative interactions and exchanges of ideas.

The second finding indicates that most students understood the volume concept satisfactorily by implementing HLT-IL. However, their levels of achievement varied depending on initial skill levels. This aligns with Clements (2020) who highlights the significance of gradual exploration in inquiry-based learning. Previous studies (Bakker et al., 2015; Watson, 2001) also emphasize the importance of scaffolding for students with lower skill levels. Unlike the technology-based learning emphasized by (OECD, 2023). The present study underscores the value of concrete manipulation in developing spatial understanding. These findings reinforce the importance of differentiating instructional strategies based on students' initial skills (Hidayati, 2020).

Although this study contributed to the design of HLT-IL to enhance students' understanding of the volume of space, it has several limitations. First, the study involved a relatively small sample size. It was conducted in a single school with relatively homogeneous student characteristics, thereby limiting the generalizability of the findings to broader and more diverse educational contexts. Second, potential bias in observing student participation, such as the subjectivity of researchers in documenting student behavior, may have influenced the accuracy of data interpretation (Miles, 2014). Therefore, future studies should involve more varied populations and settings to examine the consistency and adaptability of HLT-IL implementation in geometry learning.

This study offers a structured approach to IL and guided investigations that enhance students' conceptual understanding of volume. Several practical implications emerge for classroom practice. First, teachers are encouraged to apply HLT-IL with multilevel scaffolding (Bakker et al., 2015), providing tailored support based on students' initial skills. Low-skilled students may benefit from concrete manipulatives and intensive guidance, while high-skilled students can be challenged to pose questions, explore dimensional relationships, and generalize concepts. Second, conceptual understanding can be reinforced through open-ended worksheets, structured group work, and reflective sessions. These strategies illustrate how HLT-IL can be differentiated to meet diverse student needs in geometry learning (Miles, 2014; Nursyahidah & Albab, 2021).

## CONCLUSION

Based on the findings and discussion, this study concludes that the development and implementation of Hypothetical Learning Trajectories based on Inquiry Learning (HLT-IL), which consist of five main activities and nine sub-activities, can meaningfully facilitate students' understanding of the concept of volume of space. Conceptualization and investigation activities play a central role in constructing students' spatial representations, particularly when supported by scaffolding aligned with their initial skill levels. The variation in student skills highlights the importance of implementing differentiation strategies in applying HLT-IL within geometry classrooms. Future studies are recommended to explore the implementation of HLT-IL in more diverse contexts, including variations in student backgrounds, educational levels, and school characteristics. In addition, subsequent studies could examine the application of HLT-IL to other geometry topics or compare its effectiveness with alternative instructional approaches such as problem-based learning or blended inquiry models. Developing more systematic multilevel scaffolding is also essential to provide targeted support for low-skilled students and ensure more equitable learning outcomes.

## REFERENCES

Akker, J. J. H. van den, Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). (2011). Educational design research. Routledge, Taylor & Francis group. https://www.taylorfrancis.com/chapters/edit/10.4324/9780203088364-12/design-research-learning-design-perspective-koeno-gravemeijer-paul-cobb

Amir, M. F. (2018). Pengembangan perangkat pembelajaran berbasis masalah kontekstual untuk meningkatkan kemampuan metakognisi siswa sekolah dasar. Journal of Medives. https://doi.org/10.31331/medives.v2i1.538

Amir, M. F., & Wardana, M. D. K. (2017). Pengembangan domino pecahan berbasis open ended untuk meningkatkan kemampuan berpikir kreatif siswa SD. Aksioma: Jurnal Program Studi Pendidikan Matematika, 6(2), 178. https://doi.org/10.24127/ajpm.v6i2.1015

Artigue, M., Bosch, M., Doorman, M., Juhász, P., Kvasz, L., & Maass, K. (2020). Inquiry based mathematics education and the development of learning trajectories. Teaching Mathematics and Computer Science, 18(3), 63–89. https://doi.org/10.5485/TMCS.2020.0505

Ayuningtyas, I. N., Amir, M. F., & Wardana, M. D. K. (2024). Elementary school students' layers of understanding in solving literacy problems based on Sidoarjo context. Infinity Journal, 13(1), 157–174. https://doi.org/10.22460/infinity.v13i1.p157-174

Bakker, A., Smit, J., & Wegerif, R. (2015). Scaffolding and dialogic teaching in mathematics education: Introduction and review. ZDM, 47(7), 1047–1065. https://doi.org/10.1007/s11858-015-0738-8

Chan, K. K., & Leung, S. W. (2014). Dynamic geometry software improves mathematical achievement: Systematic review and meta-analysis. Journal of Educational Computing Research, 51(3), 311–325. https://doi.org/10.2190/EC.51.3.c

Clements, D. H. (2020). Learning and teaching early math; The learning trajectories approach; Third edition. Taylor&Francis Group. https://doi.org/10.4324/9781003083528

Creswell, J. W. (2018). Research design: Qualitative, quantitative, and mixed methods approaches. Sage Journal. https://doi.org/10.1080/15424065.2022.2046231

Feriana, O. (2016). Desaian pembelajaran volume kubus dan balok menggunakan filling dan packing kelas V. Jurnal Kependidikan, 46. https://doi.org/10.21831/jk.v46i2.9709

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. Educational Studies in Mathematics 39, 111–129. https://doi.org/10.1023/A:1003749919816

Haas, L. (1995). Improving students' inquiry skills through reflection and self-regulation scaffolds. Review of Education, Pedagogy, and Cultural Studies, 17(1), 1–6. https://doi.org/10.1080/1071441950170102

Haidar, D. A., Yuliati, L., & Handayanto, S. K. (2020). The effect of inquiry learning with scaffolding on misconception of light material among fourth-grade students. Jurnal Pendidikan IPA Indonesia, 9(4), 540–553. https://doi.org/10.15294/jpii.v9i4.22973

Hidayati, F. H. (2020). Differentiated instruction in the mathematics classroom: Teachers' teaching experience in a teacher professional development. International Journal on Teaching and Learning Mathematics, 3(1), 37–45. https://doi.org/10.18860/ijtlm.v3i1.9699

Hisyam, M., Ramadhan, A., & Budiarsa, I. M. (2021). Analysis of ability to understand the concept of space building volume in elementary school students in south palu district. Jurnal Riset Pendidikan MIPA, 1. https://doi.org/10.22487/j25490192.2021.v5.i1.pp39-47

Jais, E., Anwar, A., & Rezky, R. (2023). Desain hipotesis lintasan belajar volume bangun ruang sisi datar pada sekolah menengah pertama: Berbasis teori van hiele. Jurnal Penelitian Pendidikan IPA, 9(4), 1778–1785. https://doi.org/10.29303/jppipa.v9i4.2722

Labibah, & Amir. (2022). Development of learning trajectory of perimeter and area of squares and rectangles through various tasks. Premiere Educandum : Jurnal Pendidikan Dasar Dan Pembelajaran. https://doi.org/10.25273/pe.v12i1.12121

Maisyarah, S., & Prahmana, R. C. I. (2020). Pembelajaran luas permukaan bangun ruang sisi datar menggunakan pendekatan pendidikan matematika realistik indonesia. Jurnal Elemen, 6(1), 68–88. https://doi.org/10.29408/jel.v6i1.1713

Mashuri, D. K. (2020). Pengembangan media pembelajaran video animasi materi volume bangun ruang untuk SD kelas V. 08. https://ejournal.unesa.ac.id/index.php/jurnal-penelitian-pgsd/article/view/35876

Miles, H.-G. (2014). Book review: Qualitative data analysis. A methods sourcebook. German Journal of Human Resource Management: Zeitschrift Für Personalforschung, 28(4), 485–487. https://doi.org/10.1177/239700221402800402

Mohseni, Z., Masiello, I., Martins, R. M., & Nordmark, S. (2024). Visual learning analytics for educational interventions in primary and secondary schools: A scoping review. Journal of Learning Analytics, 11(2), 91–111. https://doi.org/10.18608/jla.2024.8309

Mutaqin, E., Asyari, L., & Muslihah, N. (2019). Hypothetical learning trajectory: Whole number multiplication in primary school. Proceedings of the 1st International Conference on Business, Law And Pedagogy. https://doi.org/10.4108/eai.13-2-2019.2286153

Neno, F., Garak, S. S., & Samo, D. D. (2023). Desain pembelajaran matematika realistik konteks permainan anak lego pada materi volume kubus dan balok untuk siswa kelas V SD. Haumeni Journal of Education, 3(1). https://doi.org/10.35508/haumeni.v3i1.10820

Nugraha, M. A. P., Sinolungan, J. S. V., Nur, R., Nuridah, S., & Cahyono, D. (2023). Conceptual analysis of problem-based learning model in improving students critical thinking skill. Journal of Education Research. https://doi.org/10.37985/jer.v4i2.185

Nursyahidah, F., & Albab, I. U. (2021). Learning design on surface area and volume of cylinder using indonesian ethnomathematics of traditional cookie maker assisted by geogebra. Mathematics Teaching Research Journal, 13(4). https://eric.ed.gov/?id=EJ1332350

OECD. (2023). Teaching for the Future: Global Engagement, Sustainability and Digital Skills. OECD. https://doi.org/10.1787/d6b3d234-en

Pedaste, M., Mäeots, M., Siiman, L. A., De Jong, T., Van Riesen, S. A. N., Kamp, E. T., Manoli, C. C., Zacharia, Z. C., & Tsourlidaki, E. (2015). Phases of inquiry-based learning: Definitions and the inquiry cycle. Educational Research Review, 14, 47–61. https://doi.org/10.1016/j.edurev.2015.02.003

Putri, H. E. (2019). Influence of concrete pictorial abstract approach to the improvement of spatial sense ability of elementary school students. Journal of Physics: Conference Series, 1157, 042083. https://doi.org/10.1088/1742-6596/1157/4/042083

Rahman, N. D., & Halim, A. N. D. (2024). The effects of inquiry-based learning environment with augmented reality integration on spatial reasoning for topic of space. Sains Humanika, 16(3), 89–99. https://doi.org/10.11113/sh.v16n3.2162

Rahmatin, N., Pramita, D., Sirajuddin, S., & Mahsup, M. (2019). Pengembangan modul pembelajaran bangun ruang dengan metode creative problem solving (CPS) pada siswa kelas VIII SMP. JTAM (Jurnal teori dan aplikasi matematika), 3(1), 27. https://doi.org/10.31764/jtam.v3i1.760

Rofieq, A. (2008). Teknik pemberian skor dan nilai hasil tes. https://ngadimunhd.wordpress.com/wp-content/uploads/2012/06/6-pemberian-skor-dan-nilai-hasil-tes.pdf

Rozgonjuk, D., Kraav, T., Mikkor, K., Orav-Puurand, K., & Täht, K. (2020). Mathematics anxiety among STEM and social sciences students: The roles of mathematics self-efficacy, and deep and surface approach to learning. International Journal of STEM Education, 7(1), 46. https://doi.org/10.1186/s40594-020-00246-z

Sahria, Y., & Yulfihani, I. (2023). Pemanfaatan teknologi augmented reality dengan metode marker based tracking sebagai media pengenalan bangun ruang. J-Sisko Tech (Jurnal Teknologi Sistem Informasi dan Sistem Komputer TGD), 6(1), 115. https://doi.org/10.53513/jsk.v6i1.7395

Sanwidi, A. (2020). Aplikasi wingeom dan media lectora inspire sebagai media pembelajaran pada materi bangun ruang geometri. Briliant: Jurnal Riset dan Konseptual, 5(4), 731. https://doi.org/10.28926/briliant.v5i4.548

Saputra, R., Novaliyosi, N., Syamsuri, S., & Hendrayana, A. (2024). Systematic literature review: Strategi scaffolding dalam pembelajaran matematika untuk meningkatkan pemahaman siswa. Jurnal Cendekia : Jurnal Pendidikan Matematika, 8(2), 1697–1710. https://doi.org/10.31004/cendekia.v8i2.3312

Seah, R. T. K., & Horne, M. (2020). The influence of spatial reasoning on analysing about measurement situations. Mathematics Education Research Journal, 32(2), 365–386. https://doi.org/10.1007/s13394-020-00327-w

Simon, M. (Ed.). (2020). Encyclopedia of mathematics education. Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0

Sisman G.T, & Meral A. (2016). A study on sixth grade students' misconceptions and errors in spatial measurement: Length, area, and volume. International Journal of Science and Mathematics Education, 14(7), 1293–1319. https://doi.org/10.1007/s10763-015-9642-5

Suciana, F., Musdi, E., & Arnawa, I. M. (2020). Pengembangan alur belajar berbasis realistic mathematic education (RME) pada materi lingkaran. Aksioma: Jurnal Program Studi Pendidikan Matematika, 9(2). https://doi.org/10.24127/ajpm.v9i2.2769

Ticu, L. (2023). Studierea geometriei gimnaziale prin conținuturi practice aplicative în clasa a VI-a. Proceedings of The Third International Scientific Conference, 174–177. https://doi.org/10.46727/c.steam-2023.p174-177

Triwahyuningtyas, D., & Suastika, I. K. (2019). Teaching "surface area of a sphere and volume of a ball" using an inquiry approach. Journal of Physics: Conference Series, 1402(7), 077103. https://doi.org/10.1088/1742-6596/1402/7/077103

Tunimah, L., Nursyahidah, F., & Albab, I. U. (2024). Pengembangan hypothetical learning trajectory materi kerucut berkonteks tradisi sesaji rewanda menggunakan PMRI berbantuan adobe animate. Aksioma: Jurnal Matematika dan Pendidikan Matematika, 15(1). https://core.ac.uk/download/pdf/609689878.pdf

van Bemmel, R., Zitter, I., & de Bruijn, E. (2023). Exploring the concept of a responsive curriculum in teacher education from the perspective of students and teacher educators. https://doi.org/10.1002/curj.295

Watson, A. (2001). Instances of mathematical thinking among low attaining students in an ordinary secondary classroom. The Journal of Mathematical Behavior, 20(4), 461–475. https://doi.org/10.1016/S0732-3123(02)00088-3

Wijaya, A., Elmaini, E., & Doorman, M. (2021). A learning trajectory for probability: A case of game-based learning. Journal on Mathematics Education, 12(1), 1–16. https://doi.org/10.22342/jme.12.1.12836.1-16